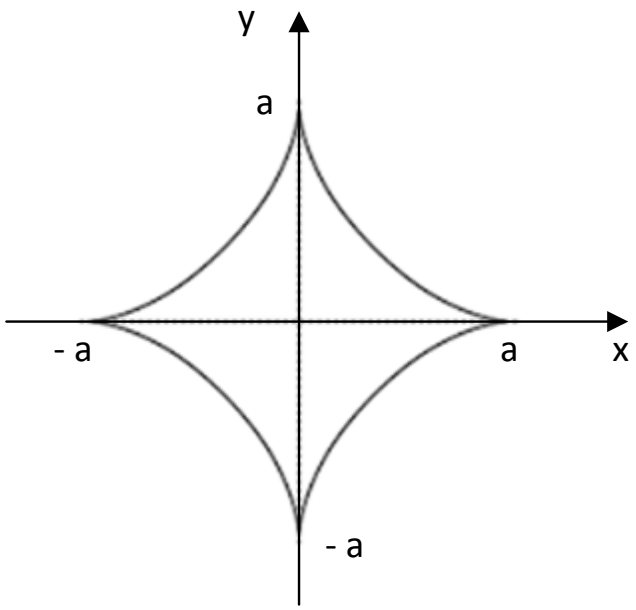
APPLIED MATH WRITTEN EXAM

1. Find the area enclosed by the astroid, $x^{2/3} + y^{2/3} = a^{2/3}$.



2.

(a) Suppose y is a function of t, find the general solution of the ordinary differential equation:

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = \cos(t)$$

(b) Suppose f is a function of t , solve the initial value problem:

$$\frac{d^2f}{dt^2} - 6\frac{df}{dt} + 9f = 0, \text{ when } f\big|_{t=0} = 1 \text{ and } \left.\frac{df}{dt}\right|_{t=0} = 2$$

3. Consider the initial value problem $y = \varphi(t)$ given by

$$y' = 4 - 7t + 6y$$

with $y(0) = 11$.

Use the classical 4th order Runge-Kutta method to calculate approximate values of the solution y of the initial value problem $y = \varphi(t)$ for values t=0, t=0.3 and t=0.6.

4. Assume that the earth is a perfect sphere and assume that gravity has no impact on the path of light. A laser beam is emitted by a laser source in Paris, perfectly parallel to the ground (i.e. perfectly horizontal). A compass is used to ensure that the laser beam is directed to the West.

   a) Calculate the distance between the laser beam and the city of New York, assuming that the radius of the earth is 6500 km and that Paris is located at 49 degrees North and 2 degrees East, while New York is located 40 degrees North and 74 degrees West. Note: X degrees North is the angle measured from the Equator, the Equator being at 0 degrees North and the North Pole being at 90 degrees North. Y degrees East or West is measured from the Greenwich meridian line.

      Please provide a drawing of problem geometry and describe your approach step-by-step before getting into detailed numerical calculation.

   b) The distance calculated above, is the distance to a certain point along the laser beam, which is not necessarily visible as it may be hidden behind the horizon. Calculate the distance between New York and the nearest visible point of the laser beam in New York, if there are no buildings, trees or hills blocking the view.

      Please describe your approach step-by-step before getting into detailed numerical calculation.