COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Fall 2016

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- All questions in this exam have a common theme: ***Hurricane Matthew***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

Suppose that you are a design engineer in a shelter equipment manufacturing company. Your current task is to design a portable shelter for hurricane season.

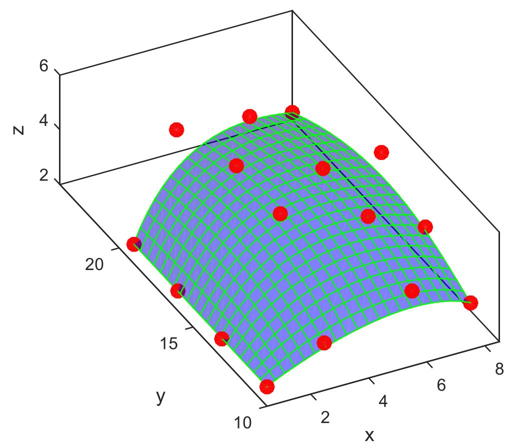
You want to use a bi-cubic Bézier surface patch to model the roof of the shelter.

a) Derive the equation of the bi-cubic Bézier surface patch in a matrix form.

b) You received the design of one patch shown on the right with the control points as

Patch 1 control points:

(1 20 2)	(3 21 5)	(6 22 4)	(8 23 3)
(1 17 2)	(3 17 6)	(6 17 5)	(8 17 5)
(1 14 2)	(3 14 6)	(6 14 5)	(8 14 4)
(1 11 2)	**(3 11 3)**	(6 11 4)	(8 11 3)



To start the design of the second patch, we have decided some of the control points as follows.

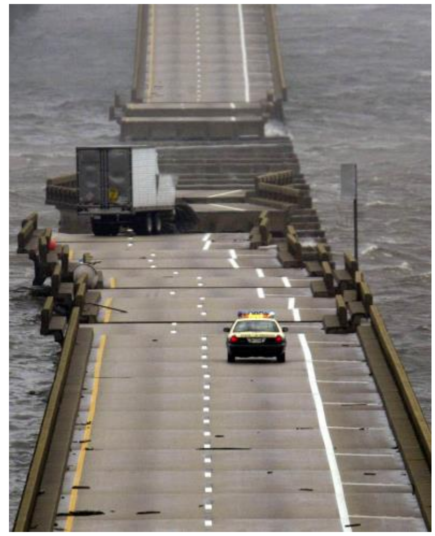
Patch 2 control points:

(-7 21 -5)	(-3 22 -4)	$\mathbf{P}_3$	(1 20 2)
(-7 17 -6)	(-3 17 -5)	$\mathbf{P}_4$	$\mathbf{P}_1$
(-7 14 -6)	(-3 14 -5)	$\mathbf{P}_5$	$\mathbf{P}_2$
(-7 11 -3)	(-3 11 -4)	$\mathbf{P}_6$	(1 11 2)

To ensure the G^0 and G^1 continuity between the two patches, what are the coordinate values of $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$, $\mathbf{P}_5$, and $\mathbf{P}_6$? Explain how you decide the values in detail and show calculations.

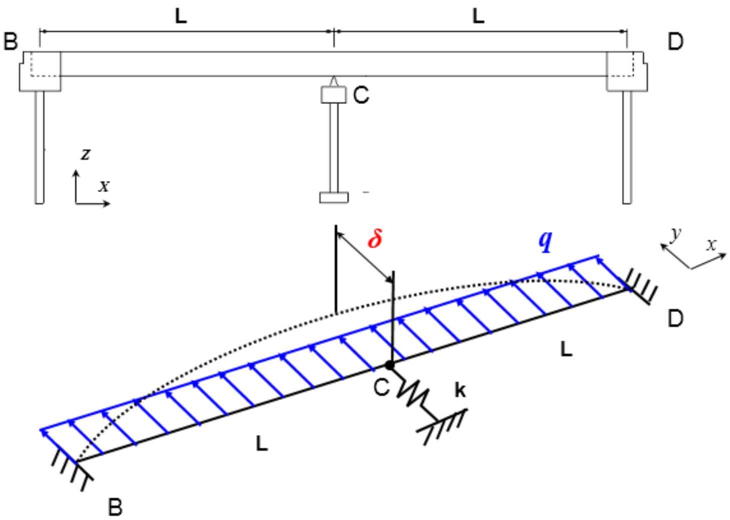
c) If you change a different control point with coordinates (**3, 11, 3**) in Patch 1 to the position (**3, 11, 2**), explain (1) how this change will affect the shape of Patch 1 and why; (2) how this change will affect the shape of Patch 2 and why; and (3) how this change will affect the continuity between the two patches and why.

Question 2 – Finite-Element Analysis



Hurricane Matthew rolled into Savannah, GA in Oct. 7, 2016 and a two span bridge was partially broken.

In the simplified model shown in the figure below, each span has a length of L, a cross-section area of A, and a moment of inertia I. The bridge is made of a material with a modulus of elasticity E. The bridge is rigidly clamped on the both ends, B and D. For simplification purpose, the partially damaged support, C, is modeled with a spring in y-direction with a stiffness of k as shown in the figure. Also, the wind force from the hurricane is assumed to be a horizontal, uniformly distributed loading, q, in y-direction.



You are asked to analyze the structure using finite-element formulation.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down the element stiffness matrix and assembly stiffness matrix.
5. Determine the y-directional deflection, δ, at C. Show all steps to find results.
6. Now assume that the support C has additional damages and the assumption of the spring support is no longer valid. In this case, the support C can be considered as a frictionless joint such that the y-directional deflection at the joint node of the two spans will be identical, but rotations will be independent. Determine the deflection, δ. Show all steps to find results. No need to calculate the final values.

<u>Element A - Stiffness Matrix</u>

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

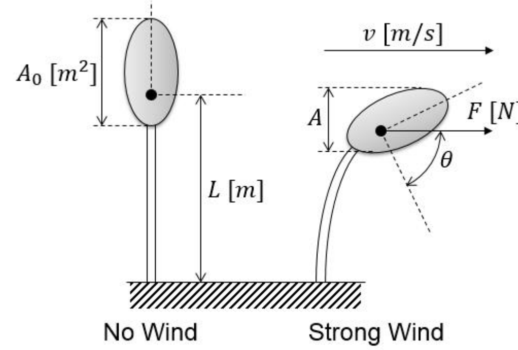
where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; l and m are direction cosines.

<u>Element B - Stiffness Matrix</u>

$$[K]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & -6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

where E, I, and L are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively.

Question 3 – Numerical Analysis



A severe hurricane has been forecast, and you worry whether the palm trees on your beach-front property will survive the strong winds. Before you commit to the expense of reinforcing the trees with cables, you decide as a proud hell-of-an-engineer to analyze the problem by relying on your knowledge of mechanics and numerical methods.
You decide to model the problem according to the diagram on the right.
When there is no wind, the crown of the palm tree has a cross-sectional surface area A_0 at a height of L. Due to the strong winds at velocity v, the trunk of the tree bends, causing the crown to tilt by an angle θ and reducing the cross-sectional surface area to $A = A_0 \cos\theta$.
A first step in your analysis is to determine the magnitude of the force, F, acting on the crown of the tree. Eventually you plan to use this force to determine whether the tree is strong enough to withstand the wind force, but for this exam problem, we will the scope of the analysis to just the computation of the force.
The force is characterized by the equation for fluid-dynamic drag: $F = \frac{1}{2} C_D \rho A v^2$, with C_D the drag coefficient, ρ the density of air, A the cross-sectional area, and v the velocity of the wind. By treating the trunk of the tree as a constant-diameter cylindrical beam under large deflection, we can approximate the bending angle as: $\theta = \frac{\alpha}{2} - 0.05\alpha^3$, where $\alpha = \frac{FL^2}{EI}$.
Before you start your computation of the force, F, answer the questions below:
3.1) From the perspective of numerical methods, what type of problem is this?
3.2) Name two numerical methods that can be used to solve this type of problem.
3.3) Discuss the advantages and disadvantages of these two methods.
3.4) Which numerical method would best be used to solve the problem listed above? Why?
3.5) Now, solve for the force, F, using the method of your choice. Find the solution with a relative accuracy of 0.001. Show your work.
Summary of equations:

$F = \frac{1}{2} C_D \rho A v^2 \qquad A = A_0 \cos\theta \qquad \theta = \frac{\alpha}{2} - 0.05\alpha^3 \qquad \alpha = \frac{FL^2}{EI}$

Values for the problem parameters (the values have been rounded to simplify the calculations):

$C_D = 0.1$	$\rho = 1.25\ kg/m^3$	$A_0 = 32\ m^2$	$v = 50\ m/s$
$L = 20\ m$	$E = 10\ GPa$	$I = 1 \times 10^{-4}\ m^4$	