Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2017

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

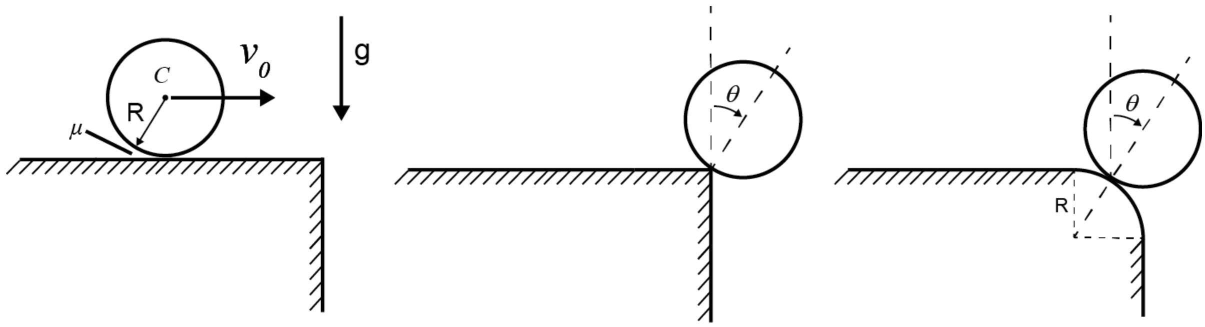
Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

A cylinder of mass m and radius r is rolling without slipping on a flat surface. The velocity of the center of mass of the cylinder is $v_0 = \sqrt{rg/3}$ when it is on the table, as shown in panel (A) below. The cylinder then reaches an edge and starts rolling off the table as shown in panel (B).

(a) Assume that there is very high static friction, $\mu \to \infty$, which prevents the cylinder from slipping on the sharp edge in panel (B). **Find the angle**, θ , at which the cylinder loses contact with the edge of the table. **Also find the angular velocity** of the cylinder at this instant.

(b) If the sharp edge of the table is instead changed to have a radius, R , as depicted in panel (C), how do the angle and angular velocity of the cylinder change when the cylinder loses contact with the rounded table?



(A) (B) (C)

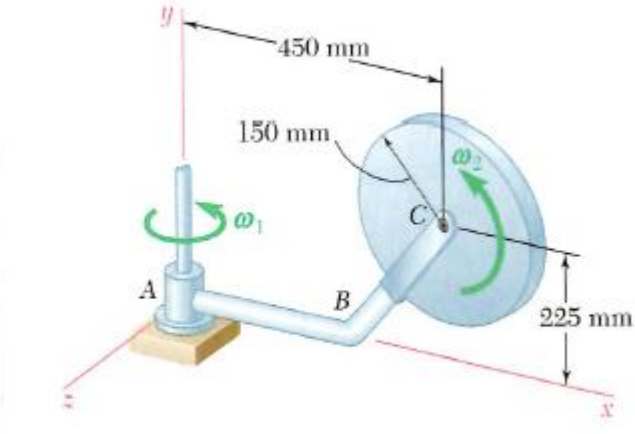
Problem 2.

A thin disk of mass 4 kg rotates with an angular velocity ω_2 with respect to arm *ABC*, which itself rotates with an angular velocity ω_1 about the *y*-axis. Knowing that $\omega_1 = 5$ rad/s and $\omega_2 = 15$ rad/s, and that both are constant,

(a) Determine the force-couple system representing the **dynamic** reactions at the support *A*.

(b) Discuss (no numbers) the additional reactions at *A* if either ω_1 or ω_2 is not constant.

Neglect the mass of *ABC* in comparison to the disk. Do not report static reactions.



Problem 3.

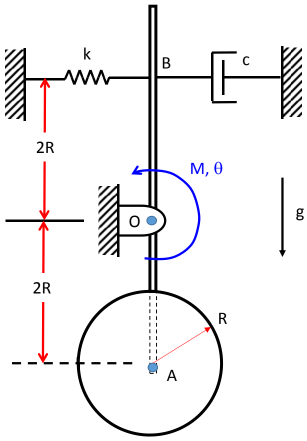
A pendulum system is shown below consists of a massless rigid rod AB which is pinned to ground by means of a frictionless pin at point O. A thin disk of mass m, radius R, and mass moment of inertia $(mR^2)/2$ is attached to the rod by means of a frictionless pin at point A. The rod is constrained by means of a horizontal spring and dashpot as shown.

(a) Find the natural frequency and damping ratio of the system.

(b) Consider a harmonically varying moment applied to rod AB, $M = Y\sin(\omega t)$. **Sketch** the amplitude response of the angle θ as a function of frequency (assume light damping). Give the high and low frequency limits.

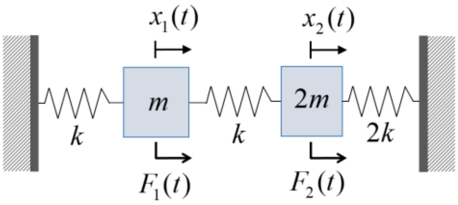
(c) Give the amplitude of the response angle θ when the frequency of the harmonic moment exactly matches the natural frequency from part (a). Express your answer in terms of the system parameters m, c, k, R, and gravity, g.

(d) Now consider the case where the disk is "welded" to rod AB so that the disk and rod must rotate together. Discuss qualitatively how your answers to parts (a), (b) ,and (c) would change. Justify your answers.



Problem 4.

Consider the following undamped 2-DOF system.



(a) Express the forced equations of motion in matrix form.

(b) Analyze the system for free vibrations, i.e. $F_1(t) = F_2(t) = 0$. Obtain the natural frequencies (in terms of *k* and *m*) and mode shapes.

(c) Let $F_1(t) = F \cos \omega t$ and $F_2(t) = 2F \cos \omega t$. Obtain the forced response (i.e. particular solution) $\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix}$ by modal analysis (do <u>not</u> use matrix inversion).

(d) Sketch the magnitude frequency response $|\, X_1 \,|$ vs. ω for the excitation in part (c). Clearly indicate the details (i.e. resonance behavior, etc.).

(e) Discuss how you would obtain a frequency response for the force in the middle spring (that connects the two masses) and comment on its magnitude vs. frequency behavior.