Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2018

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

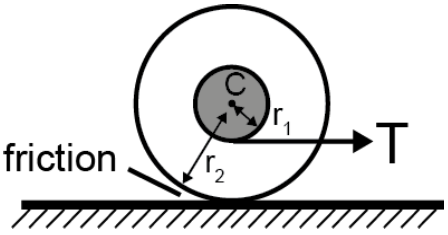
Problem 1.

A cord is wrapped around the inner drum of a wheel and pulled horizontally to the right with a force T . The wheel has a mass m , and the coefficients of static and kinetic friction are μ_s and μ_k , respectively.

(a) Find the acceleration and angular acceleration of the wheel if it is rolling on the surface

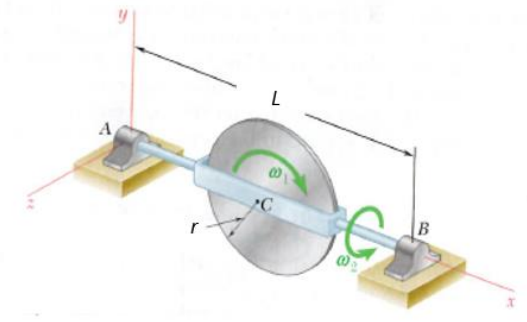
(b) Find the acceleration and angular acceleration of the wheel if it is slipping on the surface

(c) Compute the work done by the force T in bringing the drum from rest to center C velocity of v_0 for each condition above



Problem 2.

(a) The disk shown has mass *m* and spins at a constant angular speed about its center. The massless bar *AB* also spins about its axis at a constant angular speed. Determine the dynamic reactions at the bearings *A* and *B*.



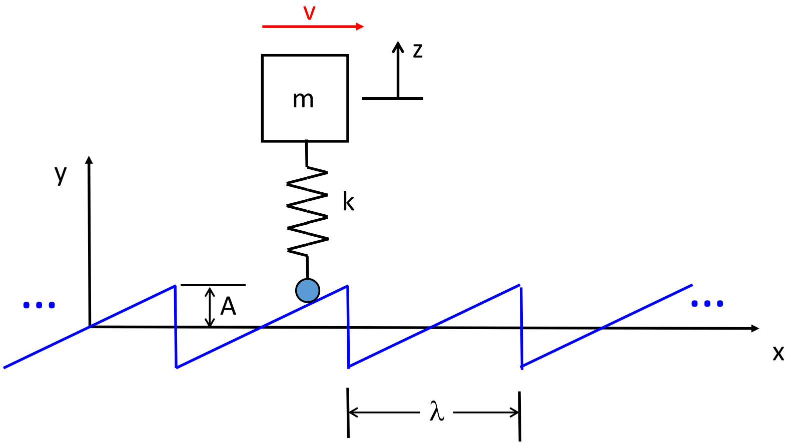
Problem 3.

A vehicle is traveling on a road surface having a sawtooth profile as shown. The forward speed of the vehicle is constant at v m/s. You are to consider a very simple, single-degree-of-freedom model of the vehicle with k = 16 N/m and m = 1 kg. The coordinate, z, is measured from the static equilibrium position of the vehicle, so gravity can be ignored. Also, you are to assume that the damping is small enough to eventually damp out the transient motion, allowing us to focus on the steady-state response. Finally, assume that the tire/spring maintains contact with the road at all times.
A Fourier Series for the road profile is provided for your convenience:

$$y(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi n x}{\lambda}\right)$$

where $$b_n = \frac{2A}{\pi n}(-1)^{n+1} \Rightarrow \quad b_1 = \frac{2A}{\pi},\ b_2 = -\frac{2A}{2\pi},\ b_3 = \frac{2A}{3\pi},\ b_4 = -\frac{2A}{4\pi},\ etc$$

(a) For a road profile with a spatial period of $\lambda = 2\pi$ meters and a forward speed of 3 m/s find a closed-form approximation for the steady-state response z(t) using 3 terms in the Fourier Series.

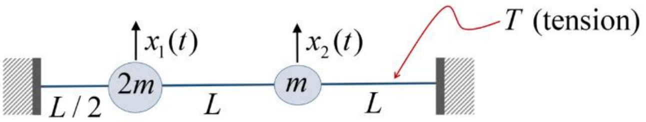


(b) From a starting speed of v = 3 m/s, by how much can the speed be increased before a critical speed is reached? By how much can it be reduced before a critical speed is reached. Note that critical speeds are characterized by violent vertical motion.

(c) Give a sketch of the "amplitude" of z(t) vs the speed v of the vehicle. Identify the locations of any peaks. You may assume that the speed is varied very slowly.

Problem 4.

The following 2-DOF system consists of a massless taut string (that has a uniform tension T and total length $2.5L$) with two point masses at fixed positions on the string. The vertical displacements of the point masses (x_1 and x_2) are measured from the given horizontal position (which is the static equilibrium). Consider small oscillations (linear vibrations).



(a) Derive the equations of motion for free vibrations.

(b) Obtain the natural frequencies (in terms of T, m, and L) and mode shapes.

(c) Sketch the mode shapes and indicate the nodal position(s).

(d) Let T = 100N, m = 1kg, and L = 1m. Obtain the response to following initial conditions

$$\begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} mm, \quad \begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$

(e) Is the response in part (d) periodic? If so, what is the period of oscillations?