GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1995

FLUID MECHANICS (FL)

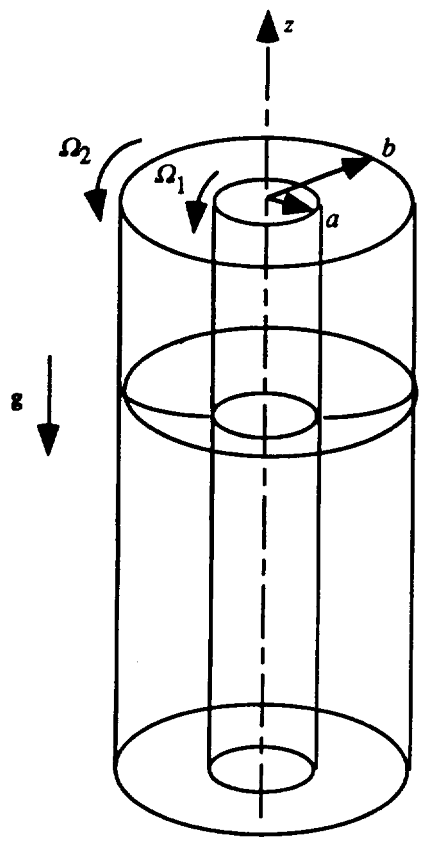
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

-- Please sign your name on the back of this page --

Work all problems

All problems are of equal weight



1. An infinitely long annulus $a < r < b$ is partially filled with a viscous, incompressible liquid, as sketched at left. Gravity is acting downward, in the negative-z direction. The two cylinders which comprise the annulus are free to rotate independently with angular speeds Ω_1 and Ω_2, respectively, as indicated.

a) Assuming the flow to be steady and rotationally symmetric, simplify the Navier-Stokes and continuity equations given below for this problem and determine both the velocity distribution in the annulus and the shape of the free surface (you may ignore surface-tension effects). State any additional assumptions which you make.

b) What would be the effect on the flow of placing a rigid, non-rotating bottom on the annulus? Without trying to solve the mathematical problem, discuss the nature of the flow in the vicinity of this bottom.

The Navier-Stokes and continuity equations in cylindrical coordinates (r, θ, z) with corresponding velocity components (v_r, v_θ, v_z) are given below:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] - \frac{\partial p}{\partial r} + \rho g_r,$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right) = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right] - \frac{1}{r}\frac{\partial p}{\partial \theta} + \rho g_\theta,$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] - \frac{\partial p}{\partial z} + \rho g_z,$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial v_z}{\partial z} = 0.$$

2. Consider a thin planar incompressible jet issuing into a quiescent semi-infinite region of the same fluid, as shown below. The fluid in the vicinity of the planar jet will be set into motion by the shear forces due to the jet. The width of the region of disturbed fluid is thin compared to the downstream (jet direction) distance; the streamlines are therefore essentially parallel to the downstream direction. The equations of motion along the downstream, or x, direction are therefore:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2};$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$$

The flow is laminar everywhere. Since these equations are essentially the same as those for a boundary layer on a flat plate (the boundary conditions are, however, different), a Blasius-type solution can be used to convert the equations of motion to a single ordinary differential equation: introduce the similarity variable

$$\eta=\frac{y}{Ax^m},$$

and let the stream function be of the form

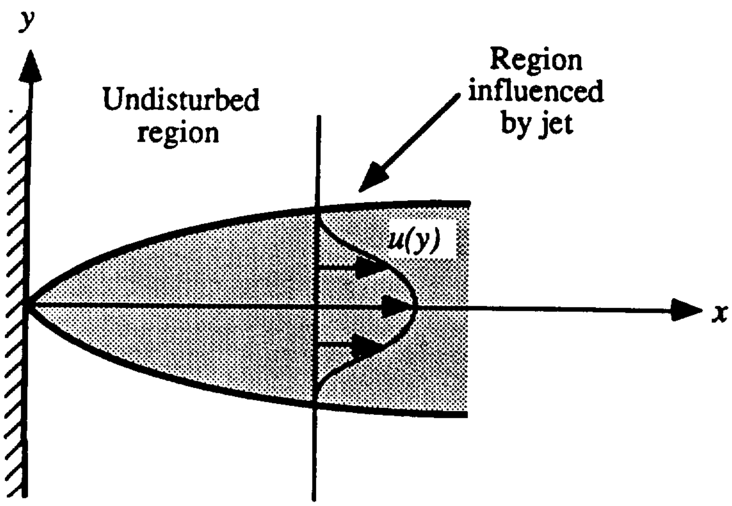
$$\Psi=Bx^n f(\eta),$$

where $A, B, m,$ and n are constants.

a) Given that the x-momentum flux should be the same through all vertical cross-sections of the flow (there are no vertical forces acting on the fluid), what is the relationship between m and n?

b) Rewrite the equations of motion as an ordinary differential equation in terms of $f(\eta)$. Use the Blasius-type solution outlined above.

c) What are the boundary conditions for this flow? Use physical arguments to justify your choice of boundary conditions.

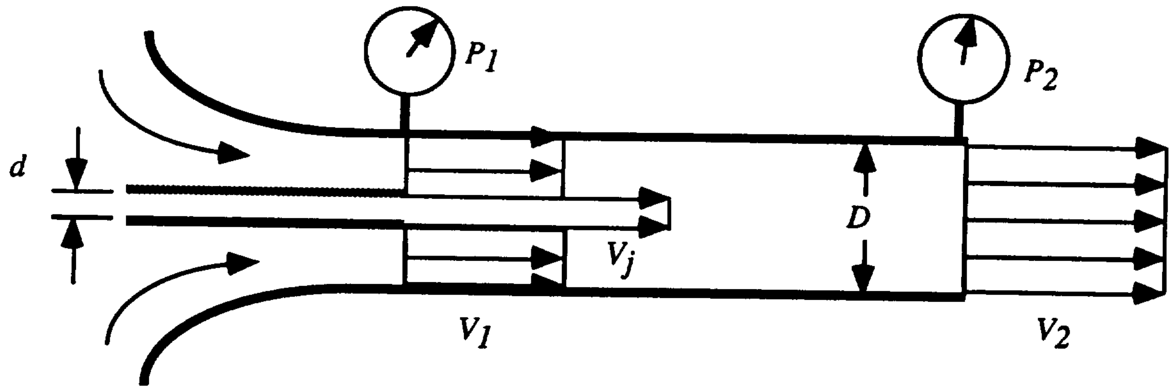


3. An injector tube of diameter d issues a water jet at velocity V_j into a mixing tube of diameter D. This results in the entrainment of water through the bell-mouthed inlet of the larger tube at speed V_1, as shown in the figure below.

a) What is the (assumed uniform) velocity V_2?

b) Assuming the pressure difference $(P_2 - P_1)$ is known, determine the shear force at the wall in terms of this pressure difference (uniform velocity profiles may be assumed for simplicity.

c) Determine the losses in terms of the pressure difference $(P_2 - P_1)$.



4. The pressure in an industrial process p_{pr} is normally monitored by a U tube mercury manometer having one side which is vented to the atmosphere. For safety reasons, it is necessary that the pressure readings will be taken remotely in the same room. Thus, it is suggested to extend the vented end of the manometer tube with a glass tube having the same diameter and length L that is filled with air and is connected to an identical manometer at its other end as shown in the sketch below.

a) Is the pressure reading of manometer "B" equal to p_{pr}? ***Explain***

b) Determine ΔH_B if ΔH_A is K.