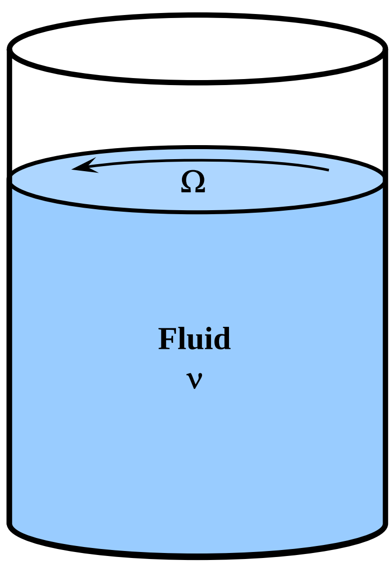
1) A cylindrical container that contains liquid is rotated about its centerline at a constant angular speed Ω. The flow has an azimuthal velocity V_θ that varies with radial coordinate r and the time after startup t; the other velocity components are zero. This velocity is a function of Ω, T, r, t, and the kinematic viscosity of the liquid ν.

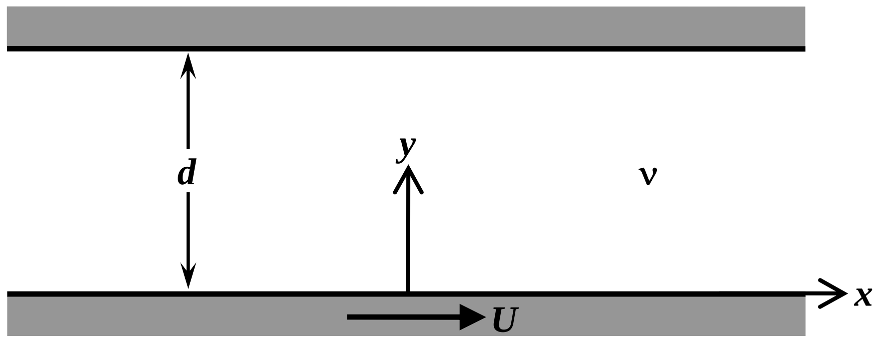
a) Determine the number of independent dimensionless groups dimensionless groups (Π terms) required to determine V_θ, and find these dimensionless groups.

b) After some startup time T, the flow becomes steady. If I have two liquids, A and B, with kinematic viscosities ν_A and ν_B, and $\nu_A > \nu_B$, how will the startup times required to reach steady-state for liquids A and B, or T_A and T_B, compare? In other words, will $T_A < T_B$, $T_A = T_B$, or $T_A > T_B$? Explain your answer (please be brief!).

c) We know that for the steady-state case, the fluid is in solid-body rotation. Show this using dimensional analysis, and comment on the implications of your result.



2)



A pair of infinitely long and parallel plates separated by a distance d have a viscous fluid of kinematic viscosity ν in the space between them. The plates and fluid are initially at rest. At time $t = 0$, the lower plate impulsively starts, moving to the right with a constant speed U. Assuming that we have only unidirectional flow along x, negligible body forces, and zero pressure gradient, we can show that the velocity is only a function of y and t. The governing equation is then the diffusion equation:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$

a) Using appropriate scales for length and velocity and the scale d^2/ν for time, *scale the governing equation* and *pose the correct boundary and initial conditions* in dimensionless form.

b) *Solve* the problem of part **(a)** by the method of *separation of variables*. You do NOT need to *evaluate* the integrals necessary for the final determination of the coefficients, but must explicitly state what they are.

c) In the absence of an upper plate (the fluid now extends to infinity in the y-direction), this problem is identical to Stokes' First Problem (also known as the "Rayleigh problem"), which has a similarity solution in terms of the dimensionless similarity variable $\eta = \dfrac{y}{\sqrt{2\nu t}}$. Determine, by substituting this variable into the *dimensional* governing equation and appropriate boundary and initial conditions, whether the problem of part **(a)** can also have a similarity solution in terms of η.

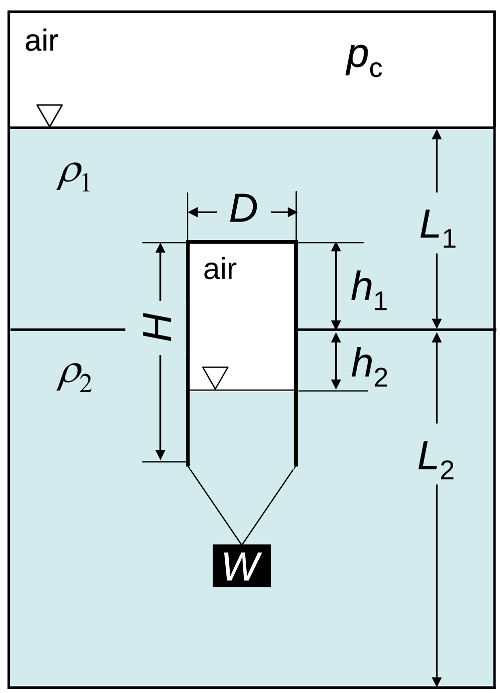
3) Two layers of immiscible liquids of densities ρ_1 and ρ_2 ($\rho_1 < \rho_2$) and the same viscosity μ, are placed in a sealed container as shown in the figure below. The air pressure above the top liquid layer p_c can be varied externally. A buoy that carries an instrumentation payload of weight W is placed near the interface between the two liquid layers. The buoy is made of a bottomless, thin cylindrical shell (diameter D and its height H) of negligible weight and wall thickness, and contains trapped air as shown in the figure. It is assumed that the presence of the buoy does not disrupt the interface between the two liquid layers.

a) Explain why and how the buoy's position (relative to the liquid interface) varies with the pressure p_c.

b) Determine the dependence of the buoy's position (relative to the interface) on the pressure p_c. Assume that the buoy position for some pressure $p_c = p_c^0$ (for which $h_1 = h_1^0$ and $h_2 = h_2^0$, etc.) is known.

c) Determine the critical pressure for which the top surface of the buoy is at the level of the interface between the two liquids.

d) Explain what happens to the buoy when p_c exceeds the critical pressure from part (c).



4) Consider a thin boundary layer formed on one side of a flat plate in free stream incompressible flow with zero pressure gradient, as shown in the figure.

a) Apply the conservation of mass and momentum to the infinitesimal control volume shown in the figure. You can adjust the height of this control volume, as you find most reasonable.

b) Use the conservation equations derived in part (a) to obtain the wall shear stress, τ_w, only in terms of the momentum thickness, free stream velocity, and fluid density.

Note: The momentum thickness, δ_t, is given by $\delta_t = \int_0^{\delta} \left(1 - \frac{u}{U}\right)\left(\frac{u}{U}\right) dy$, where U is the free stream velocity, u is the x-component of velocity profile, and δ is the boundary layer thickness.

c) Using the definition of momentum thickness, explain (briefly) the relation for the wall shear stress found in (b).