GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1996

FLUID MECHANICS

EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

-- **Please sign your name on the back of this page** --

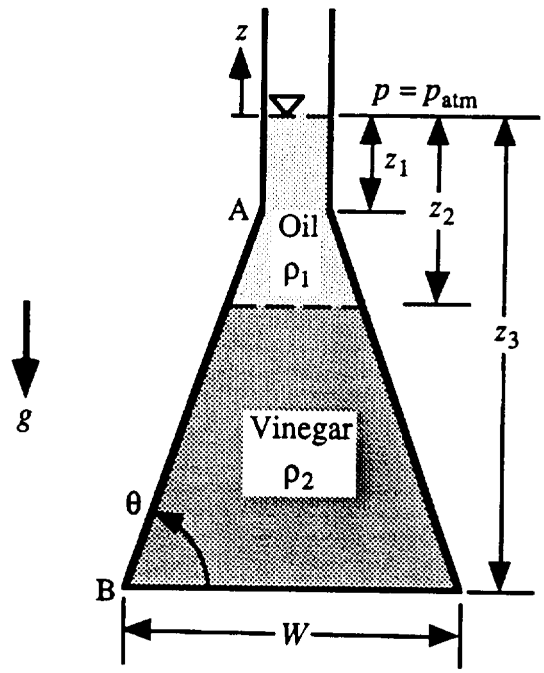
Qualifying Examination
Fluid Mechanics
Spring, 1996

Work all problems.

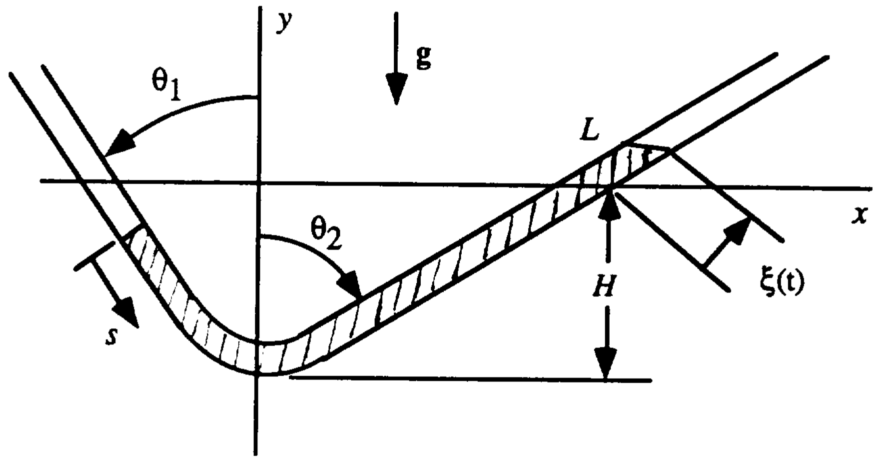
All problems are of equal weight.

1. Salad dressing consisting of oil (density ρ_1) and vinegar (density ρ_2) is stored in the tapered glass bottle shown below. Assume that the bottle has a dimension normal to the page of b.

   a) What is the pressure distribution as a function of depth, or $p(z)$ in the bottle?
   b) What is the net force $\underline{F_B}$ (magnitude <u>and</u> direction) due to the salad dressing on the bottom of the bottle, and where does it act (*i.e.*, where is the center of pressure)?
   c) What is the net force $\underline{F_S}$ due to the dressing on the side wall of the bottle AB?



2.



Compute the oscillation frequency of an inviscid liquid in an asymmetric V-tube of constant cross-section, as shown above.

[Recall that the streamwise component of the momentum equation for an inviscid, constant-density fluid with conservative body force (here, gravity) is

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial s}\left(\frac{q^2}{2} + \frac{p}{\rho} + gy\right) = 0,$$

where $q = |\mathbf{u}|$ and s is the distance in the streamwise direction.]

3. The annular gap between two coaxial circular cylinders of radii $R_1 < R_2$ contains a fluid of constant viscosity and density μ and ρ, respectively. The inner cylinder rotates about the common axis with constant angular speed Ω_1. The outer cylinder is stationary. Assume that Ω_1 is sufficiently small so that the flow is laminar (and stable), the velocity and pressure fields are axisymmetric and steady, that there is no flow in the axial direction and neglect gravitational effects:

a) Using the Navier-Stokes equations (given below), derive a differential equation for the azimulthal velocity component (list your assumptions), determine the appropriate boundary conditions, and obtain the velocity distribution within the annular gap.

b) Determine the magnitude and sign of the axial component of the torque (per unit axial length) on the *inner* cylinder.

c) Determine the magnitude and sign of the axial component of the torque (per unit axial length) on the *outer* cylinder.

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}+\frac{\partial^2 v_r}{\partial z^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right]-\frac{\partial p}{\partial r}+\rho g_r,$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{\partial^2 v_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta,$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]-\frac{\partial p}{\partial z}+\rho g_z,$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial v_z}{\partial z}=0.$$

4. Incompressible flow of an inviscid fluid of density ρ at a mean velocity U in a pipe of diameter D is accelerated around a disc of diameter d as shown. The pressure downstream of the disc is practically the same as that at the separation plane.

a) Develop an expression for the force required to hold the disc in place.
b) Calculate the force for U=10 m/s, D=5 cm, d=4 cm, and ρ =1.2 kg/m^3.