GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1997

Fluid Mechanics
EXAM AREA

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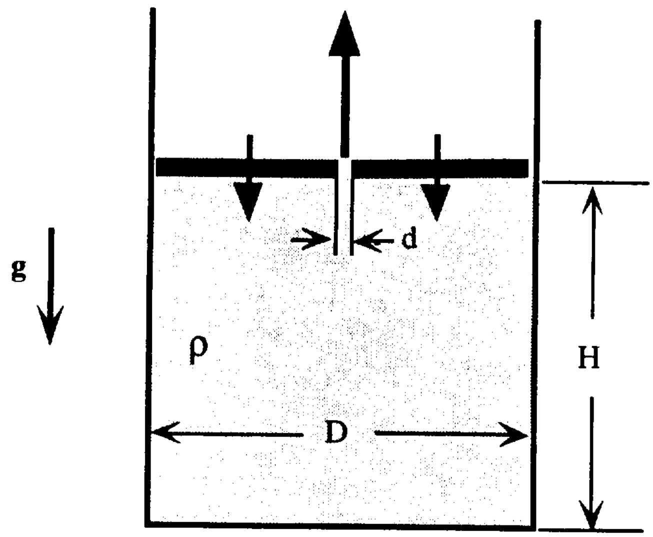
Assigned Number (DO NOT SIGN YOUR NAME)

- Please sign your name on the back of this page—

1. A vertical cylindrical container having an internal diameter D is partially filled with liquid of density ρ covered by a circular disk of weight W, as shown in the sketch below. An orifice of diameter d (d << D) at the center of the disk is unplugged, and the disk begins to drop smoothly towards the bottom of the container forcing a liquid jet through the orifice. Assuming that the disk moves with constant velocity and that the pressure at the exit plane of the jet orifice is atmospheric, determine the velocities of the disk and of the jet (at the orifice) using control volume analysis. It is also assumed that no fluid leaks through the small gap between disk and the container wall, and that the jet fluid does not accumulate on the upper surface of the disk.



2. Consider the flow of salt water past a flat plate at high Reynolds number. Besides the momentum boundary layer, a concentration boundary layer will be formed. Within the concentration boundary layer, the concentration of salt will vary from some wall value S_w to a free-stream value S_∞.

The equation governing the concentration of salt must be solved along with the momentum boundary layer equation in order to find the salt concentration profile:

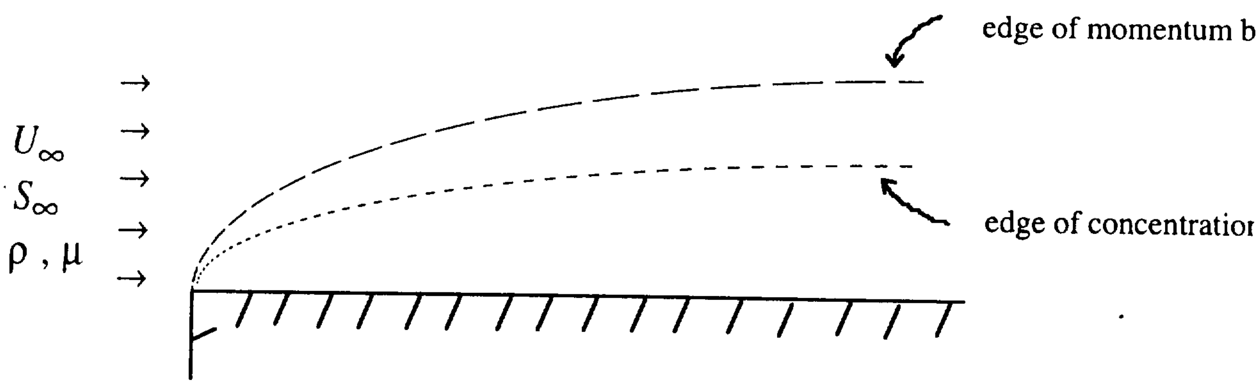
$$u\frac{\partial S}{\partial x} + v\frac{\partial S}{\partial y} = K_s \frac{\partial^2 S}{\partial y^2}$$

Where S is the salt concentration in parts of salt per million parts of water (note that S is dimensionless) and K_s is the salt diffusion coefficient, which has the dimensions of m^2/s.

i) Non-dimensionalize the above equation.

ii) What are the dimensionless parameters governing the salt concentration profile?

iii) Will the thickness of the concentration boundary layer be the same as the thickness of the momentum boundary layer? If not, what parameter will determine their relative thicknesses? Discuss the various possibilities. Use some physical reasoning here.



3. You should be familiar with the definition of a stream function for incompressible two-dimensional fluid flow in Cartesian coordinates:

$$u = \frac{\partial \Psi}{\partial y}; \quad v = -\frac{\partial \Psi}{\partial x} .$$

This definition is derived from the continuity equation representing a mass balance for a differential control volume in Cartesian coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0.$$

a) Using this form of the continuity equation, how would you define a stream function Ψ for *steady, two-dimensional* (*i.e.*, independent of z) flow?

b) Show that lines of constant Ψ are everywhere tangent to the local velocity field for steady two-dimensional flow.

c) Show that the numerical difference in Ψ between two streamlines, or $\Delta\Psi \equiv \Psi_2 - \Psi_1$, is equal to the mass flow rate per unit width passing between the streamlines for steady two-dimensional flow.

4. Consider an incompressible Newtonian fluid above an infinitely large flat plate oscillating horizontally with frequency, ω, as shown in the sketch below. The fluid in the far field is at rest, that is $u(y \to \infty, t) = 0$. The governing equation for this problem is the Navier-Stokes equation, given by:

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho \vec{V} \bullet \nabla \vec{V} = -\nabla p + \mu \nabla^2 \vec{V}$$

a) Simplify the Navier-Stokes equation for the given problem.

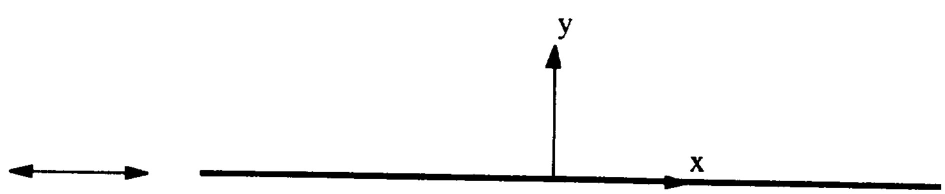
b) Considering that the form of the solution for the velocity field, $\vec{V}=(u,v,w)$, is given by:

$u(y,t) = \text{Real}\{F(y)\exp(i\omega t)\}$ and $v = w = 0$, (where $i^2 = -1$),

solve for $F(y)$ and obtain the velocity field.

c) Determine the shear stress at the wall.

d) (*bonus point*) One method to measure viscosity is to divide the force per unit area required to oscillate the plate by the shear rate at the wall. Can you suggest a different method to obtain the viscosity of the fluid knowing only the oscillation frequency, ω?



$u(y=0, t) = U \cos(\omega t)$