GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1998

Fluid Mechanics
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

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The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

1. A water container having a cross sectional area A holding water at a level h is mounted on a cart as shown in Figure 1a below. At time $t = 0$ (when $h = h_0$), water is allowed to flow out of a nozzle of cross sectional area A_j at the bottom of the tank (where $A >> A_j$). The pressure above the water level is continuously regulated so that the exit velocity of the water jet is given by $V_j = C \cdot h$ (where C is a constant).

   a) Neglecting all resistance and assuming that the weight of the cart and empty tank is negligible compared to the weight of the water, determine the acceleration of the cart for $t > 0$.

   b) At time $t = T$, a deflector is deployed in front of the nozzle as shown in Figure 1b below. Neglecting transient effects and all resistance (and assuming that the weight of the cart and empty tank is still negligible compared to the weight of the water), determine the acceleration and velocity of the cart immediately after the nozzle is deployed and at time $t = 2T$.

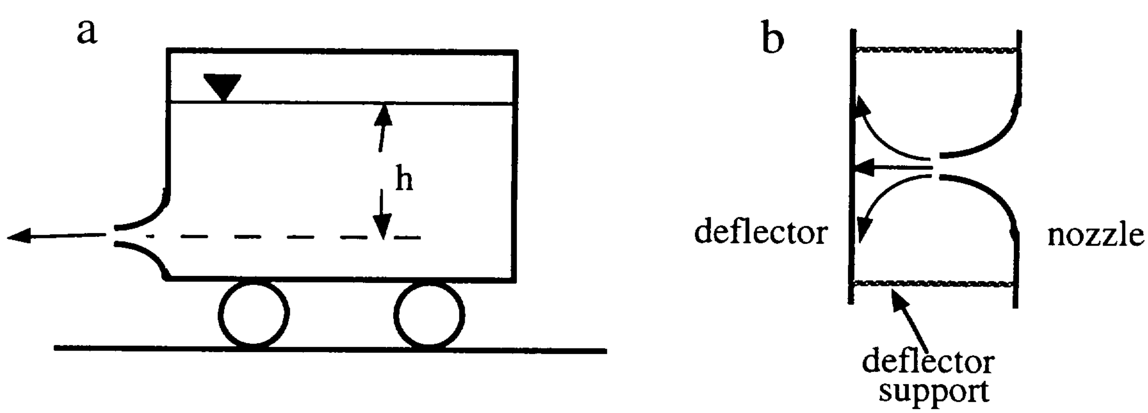


Figure 1

2. A two-dimensional, planar velocity field in a rectangular Cartesian frame is described by

$$\mathbf{v} = A\mathbf{i} + B\mathbf{j} \quad .$$

a. Sketch the streamlines if A & B are constants.

b. Obtain an expression to sketch the steamlines if $A = C_1 t$ and $B = C_2$ = constant; t represents time.

c. Use X and Y to denote the coordinates of the position of a particle at any time t, with X_0, Y_0 to denote its initial position. Obtain an expression to sketch its pathline.

d. Examine three such pathlines as indicated in part (c). The first particle is located at (0.0) at t = 0, the second is at (0,0) at t = 1 second and the third one is at (0,0) at t = 2 seconds. Sketch a streakline at t = 2 seconds that represents this description.

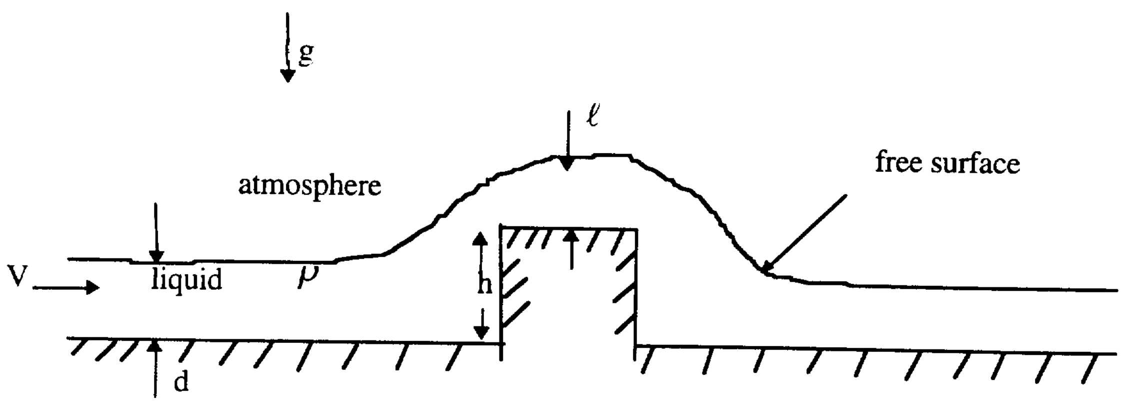
3. A layer of liquid (of depth d) flows steadily over a horizontal smooth plate, as shown below. The liquid surface is open to the atmosphere (free surface), and an obstruction of height h is placed on the plate as shown. The obstruction distorts the flow, so that the height of the free surface above the obstruction is ℓ . It is desired to do tests on a 1/10 scale model (d_m = 0.1 m, h_m = 12 m), to find ℓ and the force exerted on the obstruction, by the fluid, F. The model test is conducted in water (ρ_m = 1,000 kg/m^3, $\mu_m = 1.3 \cdot 10^{-3}$ N-sec/m^2). It is given that for the prototype flow: d_p = 1 m, V_p = 5 m/sec, h_p = 1.2 m, ρ_p = 700 kg/m^3, and $\mu_p = 3 \cdot 10^{-4}$ N sec/m^2.

a) Assuming that viscous effects are negligible, find the velocity V_m that should be used in the model test.

b) In the test of part a, ℓ_m is found to be 0.07 m. What would be the corresponding height in the prototype, ℓ_p ?

c) In the test of part a, the force on the obstruction is measured at F_m = 5N. What would be the force in the prototype, F_p ?

d) If viscous effects are much more important than the effects of gravity, what value of velocity V_m should be used in the model test?



4. The 2-D horizontal channel shown below contains two immiscible fluid layers arranged as shown. The upper plate is moving at the speed U and the lower plate is fixed. A pressure gradient in the x-direction is imposed in the channel. Determine the pressure gradient at which the interface speed between the two layers becomes zero. What is the flow rate in each liquid layer at this value of the pressure gradient?

The two-dimensional Navier Stokes equations and the continuity equation are

$$\rho\left\{\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right\}=-\frac{\partial p}{\partial x}+\mu\left\{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right\}$$

$$\rho\left\{\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right\}=-\frac{\partial p}{\partial y}-\rho g+\mu\left\{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right\}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

The shear stress in each liquid is $\tau_{yx}=\mu_i\left(u_y+v_x\right)$, where i = 1 or 2.