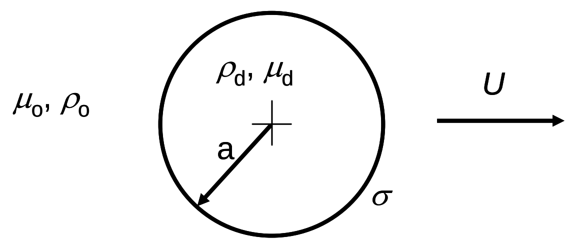
1. Consider a liquid droplet (radius $a$, density $\rho_d$, viscosity $\mu_d$) that is translating uniformly with constant velocity $U$ through another unbounded, immiscible liquid (density $\rho_o$, viscosity $\mu_o$). The interfacial tension between the two liquids is $\sigma$.

   a. Identify the dimensionless groups that describe this system and interpret their meaning by listing the ratios of characteristic forces that each group represents.

   b. Write an equation for the dependence of velocity on the dimensionless groups in part a.

   c. It is desired to conduct a dynamically similar experiment in the laboratory such that the droplet radius is 100 times larger than in the system that is being modeled. If the surface tension, density and viscosity are unchanged, how does the speed of the laboratory drop compare to the speed in the system?

   d. It is found in the laboratory experiments that gravitational effects should be considered. What additional dimensionless groups describe the relevance of gravitational forces in this system?

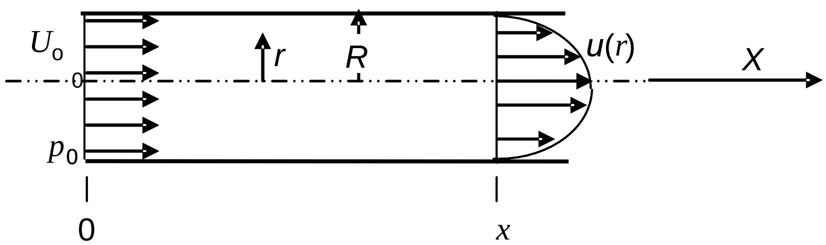


2. Consider flow of an incompressible Newtonian fluid at the entrance to a pipe, as shown in the figure. The entrance velocity profile at $X = 0$ is uniform and given by $u(r) = U_o$. Downstream, at $X = x$, the flow becomes fully developed with velocity profile $u(r) = C(r_0^2 - r^2)$, where $C$ is a constant.

   Assuming that the (static) pressure difference, $p_0 - p(X = x)$ is given,

   a. Determine the drag exerted on the pipe walls within the domain $0 < X < x$ in terms of the pipe radius $R$ and the entrance velocity $U_0$.

   b. Consider the pressure drop along a segment of the pipe in which the flow is fully developed. For constant flow rate, explain how the pressure drop changes if viscosity is gradually decreased while ***i.*** flow is still laminar, and ***ii.*** after transition to turbulent flow.



3. Consider a viscous, incompressible layer of a Newtonian fluid with the density $\rho$ and viscosity $\mu$ within a channel that is formed by two parallel plates separated by a uniform gap $h$. Each plate measures $L$ x $W$, which each dimension is much larger than $h$. The bottom plate is held stationary while the top plate is pulled to the right by a force $F_{top}$ so that it is moving with constant velocity. The flow within the channel is subjected to an *adverse* pressure gradient d$p$/d$x$.

   a. Derive an expression for the velocity distribution between the plates.

   b. Determine the force $F_{bottom}$ that is necessary to hold the bottom plate stationary. Explain the physical meaning of your result.

   c. Determine the ratio between d$p$/d$x$ and $F_{\text{top}}$ for which the net flow rate through the channel is zero.

4) A rod of constant cross-sectional area $A$ and length $L$ floats in seawater with its axis along the vertical. The density $\rho$ of the seawater varies linearly with depth $z$: $\rho(z) = \rho_o(1+Cz)$ where $C$ is a positive constant.

a. If the rod is homogeneous with a density $\rho_o$, determine the length of the rod $H$ that is submerged in seawater.

b. Is the rod hydrostatically stable? Please justify your answer.

c. What, if anything, will change if the rod now consists of two sections, each of length $L/2$ , where the bottom/lower section has a density $2\,\rho_o$ and the top/upper section has a density that is (nearly) zero?