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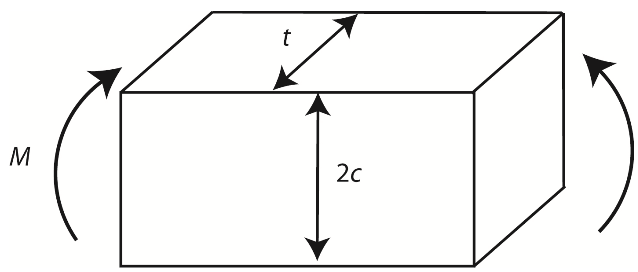
Written Examination for the PhD Qualifying Examination

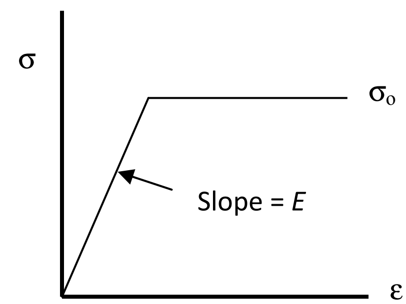
Mechanics of Materials

October 2011

Instructions: Please solve 3 of the following 4 problems. Please show all work and write your name on any additional sheets used in the solution.

1. A rectangular beam has a depth $2c$ = 30 mm and thickness t = 10 mm, and is made of an aluminum alloy having elastic, perfectly plastic stress-strain behavior (shown below), with elastic modulus E = 70 GPa, and yield stress σ_0 = 350 MPa. A bending moment M = 650 N-m is applied as shown and then removed.





(a) Determine the residual stresses, and plot their distribution as a function of the distance y from the neutral axis (from $y = -c$ to $y = c$). On the same plot, also show the stress distribution at the maximum moment.

(b) Plot the stress-strain path during loading and unloading for the upper surface of the beam ($y = c$).

(c) What happens to the beam if we attempt to apply a bending moment of 900 N-m upon initial loading?

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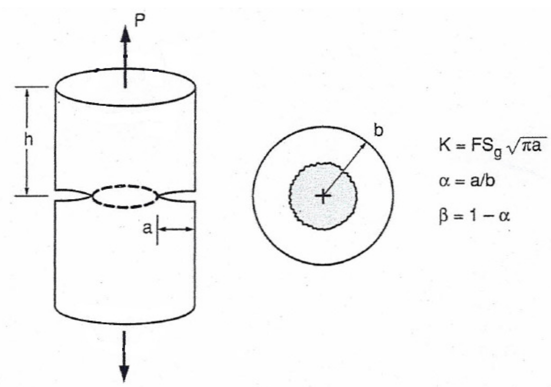
2. Tensile tests are conducted on circumferentially cracked specimens as shown in the Figure below to measure their fracture toughness. An inadvertent error during handling caused the cracks in all the specimens to be filled with an adhesive which has a tensile strength of σ_a. The cracks are sharp.

(a) Derive a relation for the fracture toughness of the specimen without adhesive by correcting for the effect of the adhesive. Clearly state your assumptions. *Hint: consider the use of superposition of solutions for this linear elastic problem.*

(b) The materials tested and their properties are listed in the Table below. If the dimensions of the specimens are $h = 100\,mm$, $b = 50\,mm$ and $a = 8\,mm$ and the adhesive strength is $\sigma_a = 20\ \text{MPa}$, do these tests provide valid measurements for these materials?



(a) Axial load P: $S_g = \dfrac{P}{\pi b^2}$, $\quad F = 1.12 \quad (10\%,\ a/b \le 0.21)$

$$F = \frac{1}{2\beta^{1.5}}\left[1 + \frac{1}{2}\beta + \frac{3}{8}\beta^2 - 0.363\beta^3 + 0.731\beta^4\right]$$

Material	K_{IC} (MPa$\sqrt{\text{m}}$)	σ_0 (MPa)
AISI 1144 steel	110	1090
2024-T351	34	352

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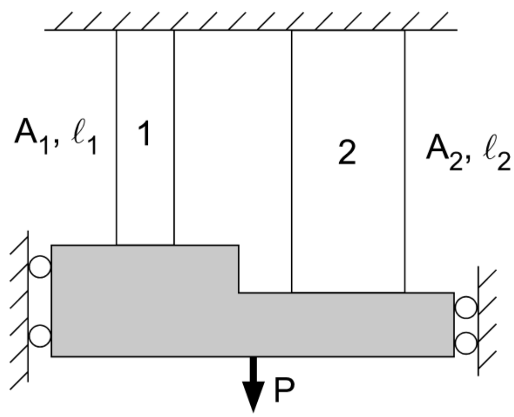
3. Deformable bars 1 and 2 shown below have respective cross sectional areas (A_1, A_2), lengths (ℓ_1, ℓ_2), elastic properties (E_1, E_2), and coefficients of thermal expansion (α_1, α_2). Ignore end effects where bars are attached to rigid bodies (ceiling and horizontal shaded fixture on rollers).



(a) Please derive the relationship between the axial stress, σ_1, and strain, ε_1, in Bar 1 in terms of the force, P, the dimensions of the bars, and the elastic modulus of Bar 2. For this part, assume the stresses in the bars are zero when P = 0 and that the temperature is constant.

(b) If P = 0 and both bars can be heated independently to temperatures T_1 and T_2, what is the relationship between the axial stress and mechanical strain in Bar 1 in terms of all other known parameters?

(c) If $A_2 >> A_1$ and Bar 1 is elastic-perfectly plastic with properties E_1 = 400,000 MPa, σ_{y1} = 800 MPa, and $\alpha_1 = 5.0 \times 10^{-6}$ 1/°C, determine the response of Bar 1 when it is heated from room temperature (25°C) to 625°C while the temperature of Bar 2 remains at room temperature.

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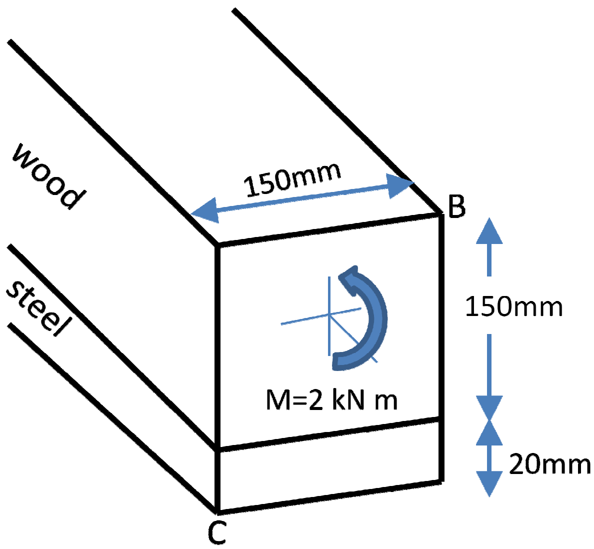
4. A composite beam made of wood is reinforced with a steel strap located on its bottom side, as shown in the Figure below. The beam is subjected to a bending moment of M = 2kN-m. The properties of the materials are given in the Table below.

Property	Wood	Steel
Young's Modulus	12 GPa	200 GPa
Tensile strength (parallel to beam length)	87 MPa	400 MPa
Tensile strength (perpendicular)	2 MPa	400 MPa
Compressive strength (parallel)	23 MPa	N/A
Compressive strength (perpendicular)	9 MPa	N/A



(a) Assign a coordinate system and calculate the location of the neutral axis.
(b) Draw the variation of the normal strain (profile) with distance from the neutral axis.
(c) Draw the variation of the bending stress (profile) with distance from the neutral axis.
(d) What is the normal stress at points B and C?
(e) What are the respective normal stresses in the wood and in the steel at their interface?
(f) Based on maximum normal stress theory, will the beam fail? If no, why? If yes, which of the two materials will fail first and at which location?

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