GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1998

Mechanics & Materials
EXAM AREA

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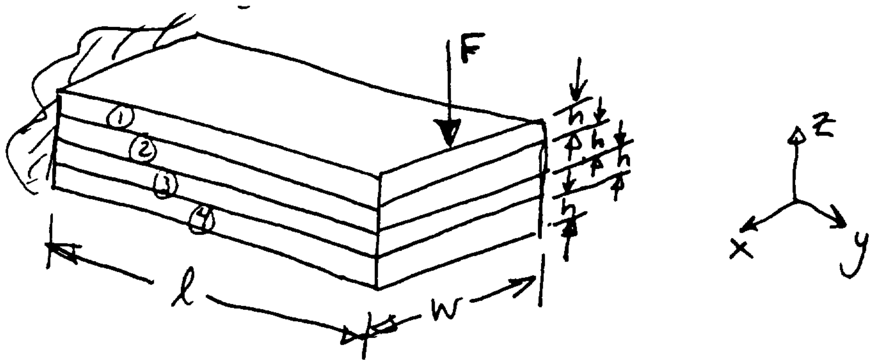
Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Please **print** your name here.

The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

1. A rectangular cantilever beam is built up from a number of layers of individual thickness, h, composed of dissimilar materials, as shown below. Each material is linear elastic, with distinct elastic constants. The layers of the beam are sufficiently wide to consider the beam in plane strain bending when subject to an end load, **F**, as shown. The beam is fixed to the wall at left. Assume that $E_1 = E_4$ and $E_2 = E_3$.



a. Suppose you don't know whether the layers are intimately bonded or are allowed to slide relative to each other. Can you make limiting assumptions and estimate an upper and lower bound of the force **F** required to produce a given (small) displacement δ at the end of the beam? Show all work.

b. What are the boundary conditions at the wall?

For parts c,d and e, assume that all layers are intimately bonded at their interfaces.

c. Are the stresses uniformly distributed in each layer at the center of the span, $\ell/2$? At the end where **F** is applied, ℓ? Explain why or why not in each case.

d. In the global x-y-z system, please indicate the nonzero components of the stress tensor in each layer.

e. Do plane sections remain plane in bending of this beam? Why or why not?

2. Consider a continuous, fiber-reinforced, unidirectional laminate composite. The Fiber properties are given as E_{fl} (logitudinal) and E_{ft} (transverse) with Poisson's ratio of ν_{12} in both directions. The Matrix material is assumed to be isotropic with the modulus E_m and Poisson's ratio ν_m.

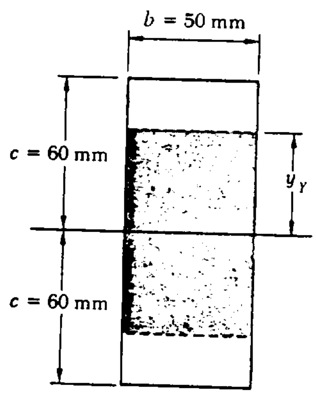
A) Derive the Rule-of-Mixtures equation for the composite's modulus in the composite's longitudinal direction (i.e. fiber direction). State your assumptions and show all work.

B) The carbon fibers have a longitudinal modulus of 60 msi and a tranverse modulus of 4msi. The fiber's Poisson's ratio is 0.25. The fiber behaves elastically to failure at a strain of 1.2%. The epoxy matrix has a modulus of 2 msi, a tensile strength of 4 ksi, a Poisson's ratio of 0.4 and is also elastic to failure. Calculate:

a. The composite's tensile modulus.
b. The composite's tensile strength.
c. The composite's strain to failure.

Qualifier Problem (Mechanics of Materials) Spring 1998

3. A member of uniform rectangular cross section 50 by 120 mm is subjected to a bending moment M = 36.8 kN-m. Assuming that the member is made of an elasto-plastic material with a yield strength of 240 MPa and the modulus of elasticity of 200 GPa determine
a) the thickness of the elastic core
b) the radius of curvature of the neutral surface.
c) the distribution of residual stresses after the bending moment has been decreased to zero.



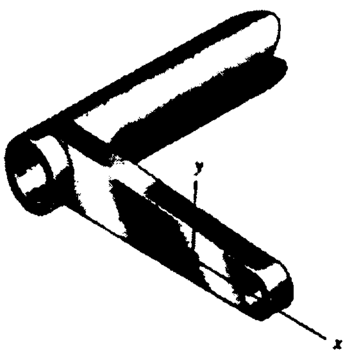
4. The state of strain at the point on the surface of the bracket has components $\varepsilon_x = -0.00013$, $\varepsilon_y = 0.00028$, and $\gamma_{xy} = 0.00008$. The material is a tool steel with $E = 200$ GPa and $\nu = 0.32$.

(a) What is the state of stress at this point with reference to the coordinate system shown? Please indicate all independent components of the stress tensor.

(b) What is the complete state of strain at this point with reference to the coordinate system shown?

(c) What are the principal stresses at this point?

(d) Please show the orientation of the principal axes on the drawing.



5. Compare (and contrast) briefly the typical plastic deformation behavior of for metals having (a) Body Centered Cubic, (b) Face Centered Cubic, and (c) Hexagonal Close Packed crystal structures.