M.E. Ph.D. Qualifier Exam
Spring Semester 2002

GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

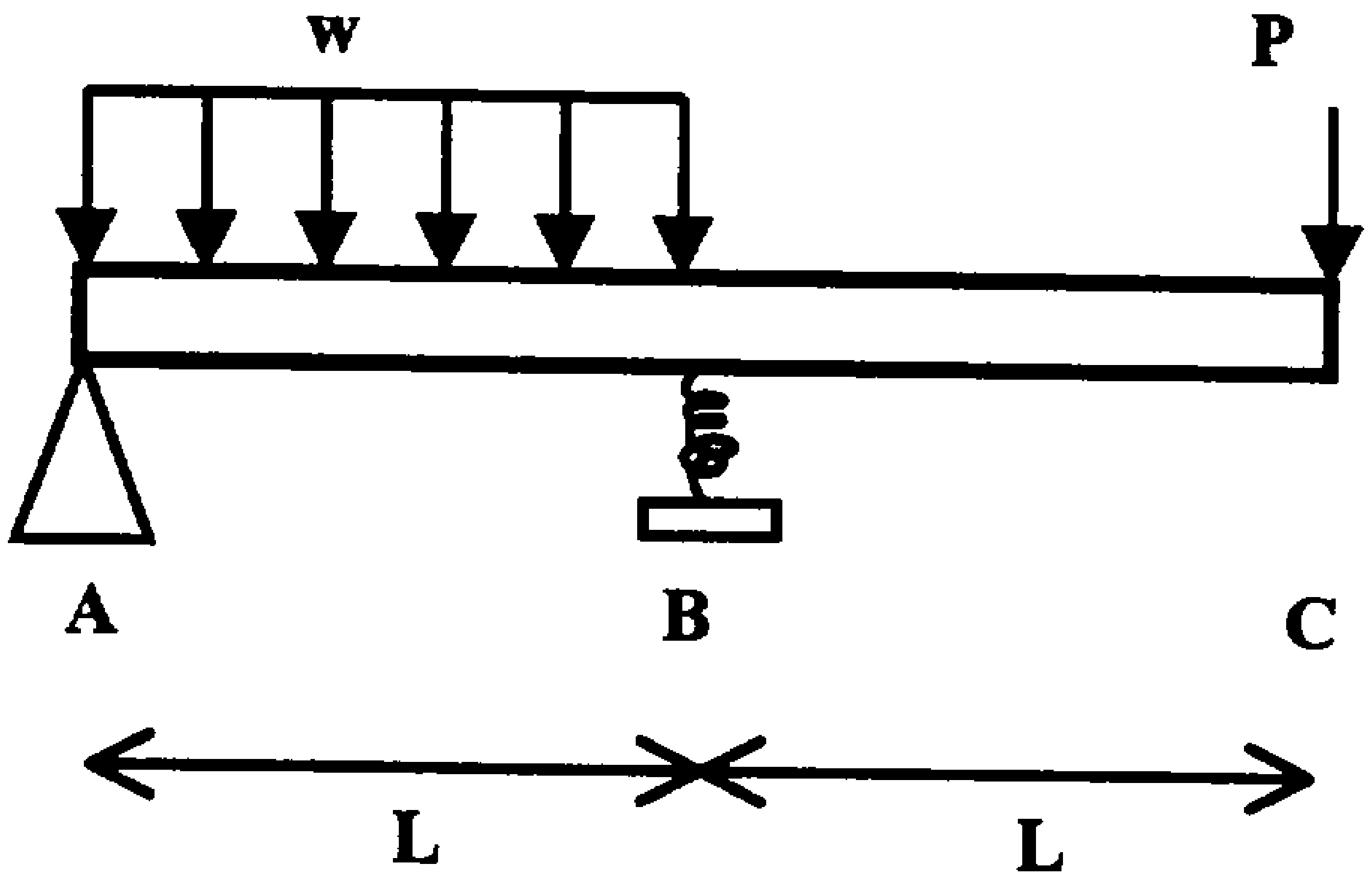
Ph.D. Qualifiers Exam - Spring Semester 2002

Mechanics of Materials
EXAM AREA

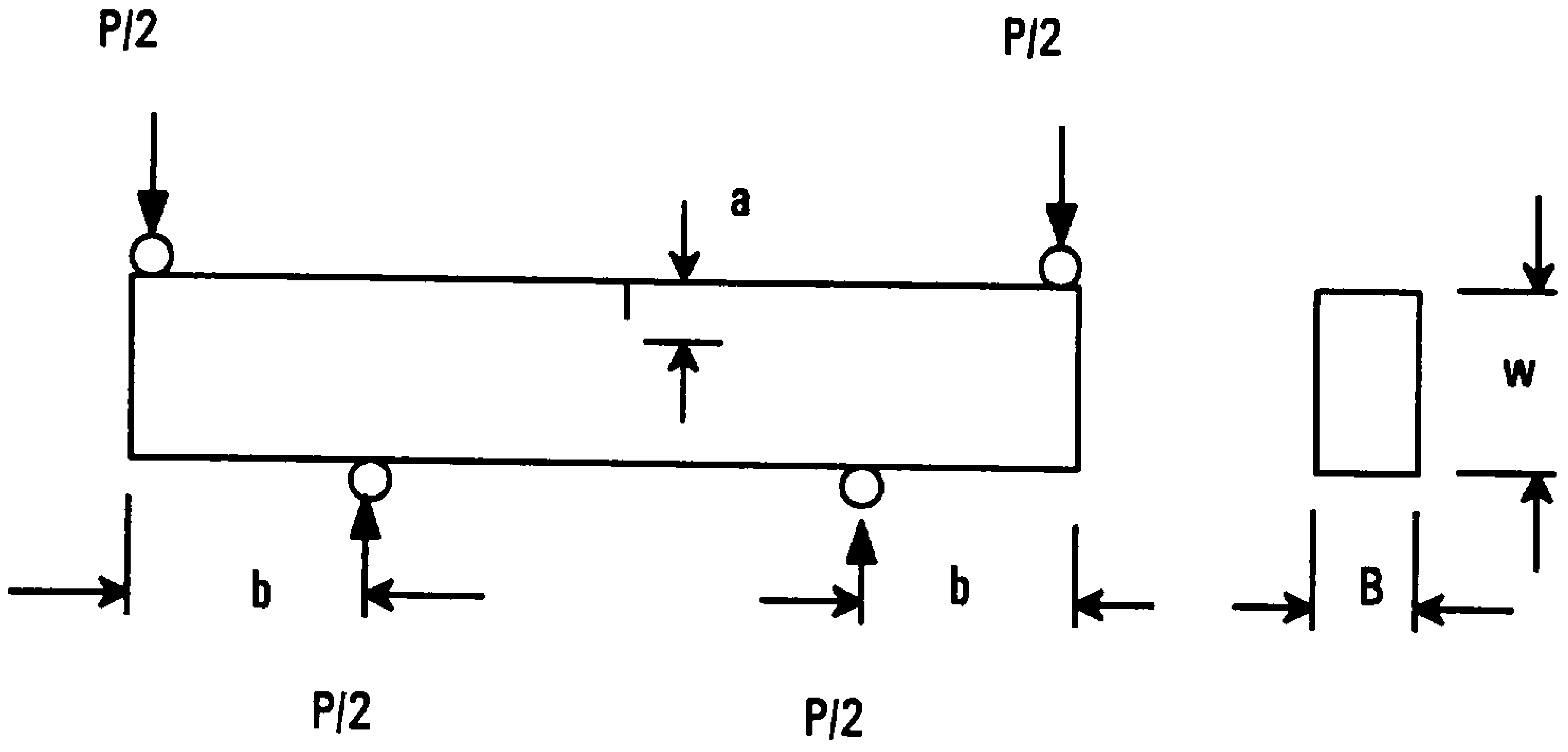
Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

Problem 1: An overhand beam rests on a simple support at A and a spring support (spring constant k) at B. The beam supports a distributed load **w** and a point concentrated force **P** at C. Solve for the deflection (δ_c) at the overhang end. Clearly state all assumptions.



Problem 2: Two specimens are cut as shown in Figure 1. Table 1 gives the dimensions and material properties.



$$K_I = 1.12\sigma^{\infty}\sqrt{\pi a}$$

Table 1

Material	Yield stress	Fracture toughness	c	m	b	B	w	a
	(MPa)	(MPa√m)			(mm)	(mm)	(mm)	(mm)
AISI 4130	1090	110	5.11x10^{-10}	3.24	10	3	4	0.5
6061-T651	275	34	2.71x10^{-8}	0.64	10	3	4	0.5

$$\frac{da}{dN} = c\Delta K^{m}$$

where crack growth rate is in mm/cycle and K is in MPa√m

1. Sketch shear force and moment diagrams.
2. Calculate the maximum stress in the beam for P=800N
3. If each beam is loaded to failure, determine the expected failure mechanism (plastic yield or fracture). Show your analysis.
4. If the beam is cyclic-loaded from 0 to 800N and the two materials follow Paris' law fatigue crack growth behavior, determine the number of cycles required for 0.1mm crack growth in steel.
5. Can Paris' law be used for the aluminum beam at this load? Why?

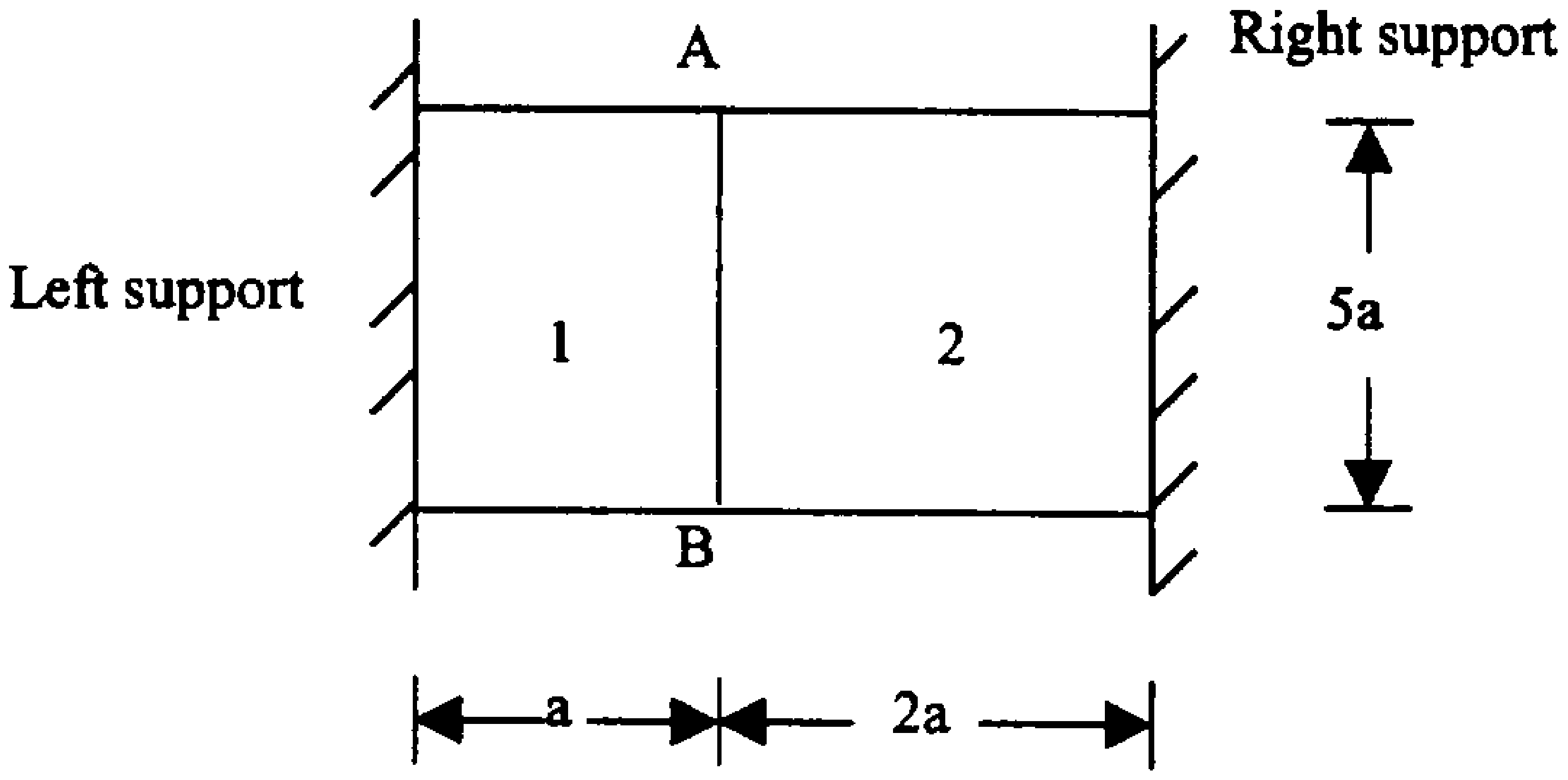
Problem 3: Shown below are two uniformly thin plates of thickness h and in-plane dimensions as shown in the figure below. The plates butt up to one another along AB. Plate 1 is made of aluminum and plate 2 of steel. The two support surfaces are rigid and fixed. Both plates are heated uniformly and experience a temperature change of ΔT. The following data is given:

$E_1 = 10^7$ psi, $\alpha_1 = 22.5 \times 10^{-6}/°F$
$E_2 = 30 \times 10^6$ psi, $\alpha_2 = 11.9 \times 10^{-6}/°F$
$a = 10"$
$h = 0.5"$
$\Delta T = 10°F$

a) What boundary conditions need to be in effect along the interface AB and the left and right supports so that the stress resulting from the temperature change is at least approximately uniform? Be very specific.

b) Determine the stress(s) in each plate.

c) What is the maximum shear stress in the plates(s)?



Problem 4: The bi-material cantilever beam below is subjected to loading as shown. Neglect transverse shear and use simple beam theory. The stress-strain curves for both materials 1 and 2 are shown given. Material 1 has symmetric yielding behavior in tension and compression, while Material 2 has twice the yield strength in compression compared to tension. Note that $\sigma_{yT1} = 2\sigma_{yT2}$. The Young's modulus of Material 2 is equal to that of Material 1, $E_1 = E_2$.

(a) Determine the purely elastic solution for the magnitude of the deflection at the end of the beam, x = L.

(b) Where will the beam first yield (x = ?, y = ?), and at what value of applied load P?

(c) Determine the limiting load that the fully plastic (fully yielded) beam can sustain.

(d) Repeat the solutions to parts (a) to (c) above if the load P at the end of the beam is applied in the opposite sense (i.e., in positive y direction).

(e) Does the condition $\varepsilon_x = 0$ hold at the wall, x = 0? Why or why not?

(f) If $E_1 > E_2$, for the elastic solution does the neutral axis remain fixed at the center of the beam, y = 0? Explain thoroughly.