1. The state of stress at a critical point in a component is

$$[\sigma] = \begin{bmatrix} 400 & 0 & 100 \\ 0 & 0 & 0 \\ 100 & 0 & 200 \end{bmatrix} \text{ MPa.}$$

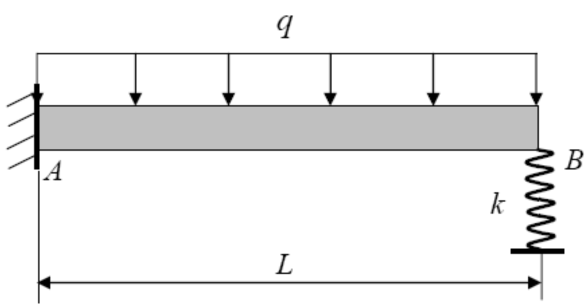
(a) Is this a state of plane stress? Explain.

(b) Is this a state of plane strain? Explain.

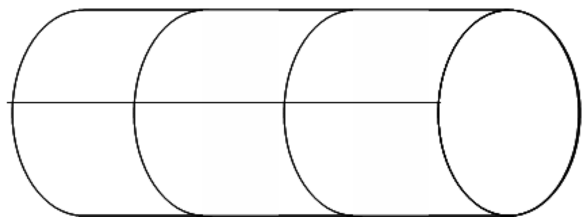
(c) What is the factor of safety against yielding if the yield strength of the material measured in a tensile test is 600 MPa?

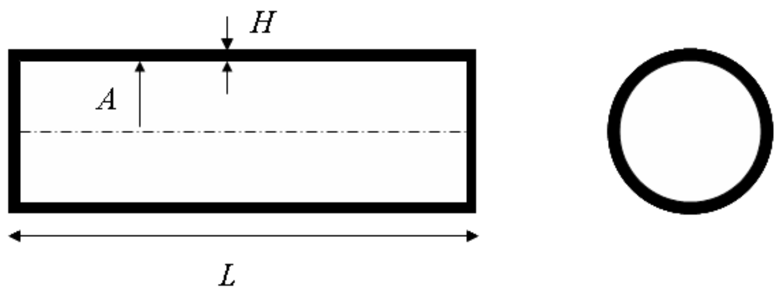
2. A cantilever beam *AB* of length *L* has a fixed support at *A* and a spring support at *B* (see figure). The spring behaves in a linear elastic manner with stiffness *k*. The bending stiffness of the beam is *EI*.

   If a uniform load of intensity *q* acts on the beam, what is the downward displacement δ_B of end *B* of the beam?



3. Consider the unloaded cylinder that is closed on both end and has seams at angles of 0- and 90-degrees from the horizontal axis. Here, *A* is the inner radius of the cylinder, *H* is the thickness of the cylinder, and *L* is the length of the cylinder in the unloaded configuration.





Unloaded Dimensions

(*i.*) Let the pressure inside the cylinder be increased quasi-statically. Assuming that the seams will fail before the material, which seam do you think will fail first and why?

(*ii.*) (a.)At a pressure *P*, derive the expression for the mean normal stresses in the circumferential and axial directions, in terms of applied loads and geometry. (You may assume that the material is linear, elastic, homogeneous, and isotropic (LEHI) and experiences small strains.) Describe, qualitatively, how your result from (*ii.*) (a.) will change if the material (b) exhibits non-linear behavior, (c) exhibits anisotropic behavior or (d) experiences large deformations?

(*iii.*) Next, at pressure *P*, let a torque *T* be applied to each end of the cylinder, as shown. Which seam do you think might fail under this loading scenario? What are the components of stress in the cylinder? (You may assume that the material is linear, elastic, homogeneous, and isotropic (LEHI) and experiences small strains.) Describe, qualitatively, how your result from (*iii.*) (a.) will change if the material (b) exhibits non-linear behavior, (c) exhibits anisotropic behavior or (d) experiences large deformations?

(*iv.*) The Lame Solution for inflation and extension of a LEHI tube are:

$$\sigma_{rr} = \frac{P_i a^2 - P_o b^2}{b^2 - a^2} - \frac{(P_i - P_o)a^2 b^2}{(b^2 - a^2)r^2} \qquad \sigma_{\theta\theta} = \frac{P_i a^2 - P_o b^2}{b^2 - a^2} + \frac{(P_i - P_o)a^2 b^2}{(b^2 - a^2)r^2}$$

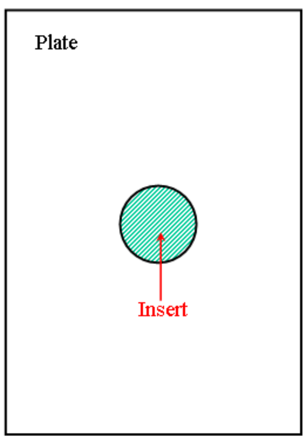
$$\sigma_{zz} = \nu\,(\sigma_{rr} + \sigma_{\theta\theta}) + \varepsilon_{zz} E$$

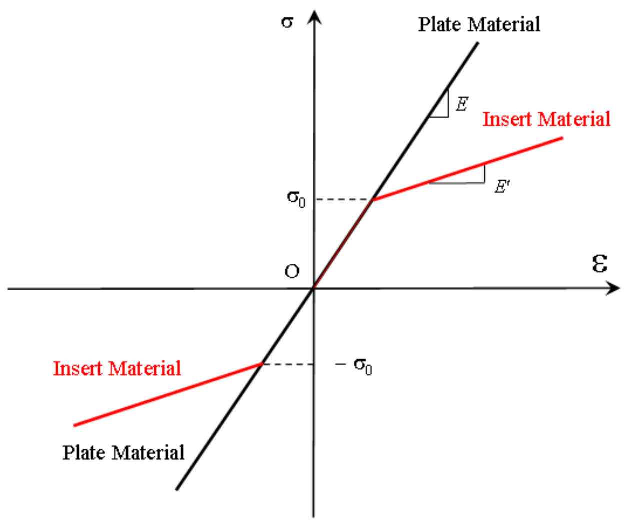
Given your results from (ii.), which (should) provide a much simpler relation for the stress in a cylinder, why did Gabriel Lame (a brilliant mathematician, physicist, chemist, etc.) arrive at this much more complicated set of solutions, even though this solution is for a simple LEHI material?

4. Consider a very large and thin plate made of a linear elastic material with a hole of radius r_i. If the surface of the hole is subjected to internal pressure p, the radial and hoop stresses in the plate are

$$\begin{cases} \sigma_r = -p\dfrac{r_i^2}{r^2}, \\ \sigma_\theta = p\dfrac{r_i^2}{r^2}. \end{cases}$$

In the above expressions, r is the radial distance from the center of the circular hole to the point of interest in the plate. For example, on the surface of the hole, $r = r_i$, $\sigma_r = -p$ and $\sigma_\theta = p$. Now, in this problem, a circular insert made of an elastic-plastic material is placed into the hole without inducing any stress because the radius of the insert matches the radius of the hole perfectly. The stress-strain relations for the materials are illustrated in the attached figure. Assume the coefficient of thermal expansion for the plate is zero and the coefficient of thermal expansion for the insert is α.





(a) If the temperature of the plate-insert assembly is gradually and steadily increased, calculate the radial stress between the insert and the plate when yielding occurs in the insert;
(b) What is the temperature increase required to cause yielding?
(c) State the radial stress as a function of temperature increase for temperatures leading up to yielding;
(d) Briefly explain in words how you would find the radial stress at the interface as a function of temperature increase after yielding has occurred, you do not have to find this relation - a short and clear explanation in words will suffice;
(e) Compare the stress tensor, strain tensor and displacement vector on the two sides of the interface between the plate and the insert. Specifically, state which components of each quantity are continuous and which components are not continuous across the interface.