GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1995

Dynamics & Vibrations
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

-- Please sign your name on the back of this page --

Dynamics and Vibrations Qualifying Exam, Fall, 1995

Work any 3 of the 4 problems. Show all of your work. Clearly state all assumptions.

A one-cylinder reciprocating engine rests on a lossless elastic foundation as shown. The total mass of the engine is M while the mass of the piston alone is m. The elastic stiffness of the foundation is k. Assume that the crankarm is massless with length e and that the connecting rod is massless with length L.

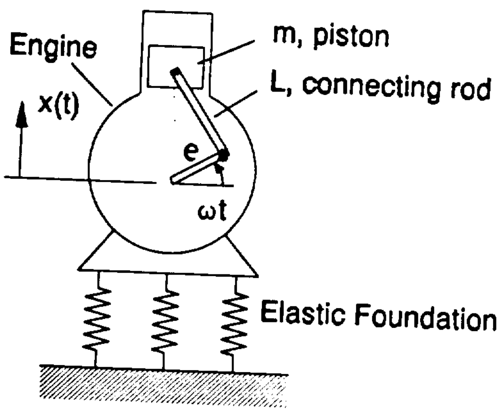
(a) Considering only the vertical motion and forces for this system, derive the equation of motion for the engine displacement x(t). If possible, simplify the equation of motion assuming that $(e/L) << 1$.

(b) Even if you could not complete part (a), consider the following approximate linear model for the engine:

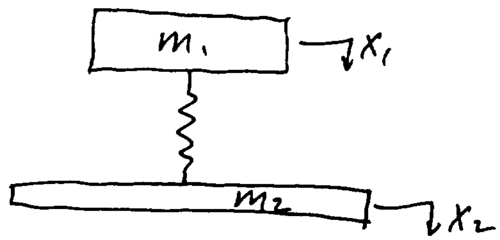
$$M\ddot{x} + kx = me\omega^2[\sin(\omega t) - \frac{e}{L}\cos(2\omega t)]$$

Find the steady-state response of the engine for the case $\omega = 2\omega_n$.

(c) In terms of ω_n, what are the systems's critical rotation speeds?



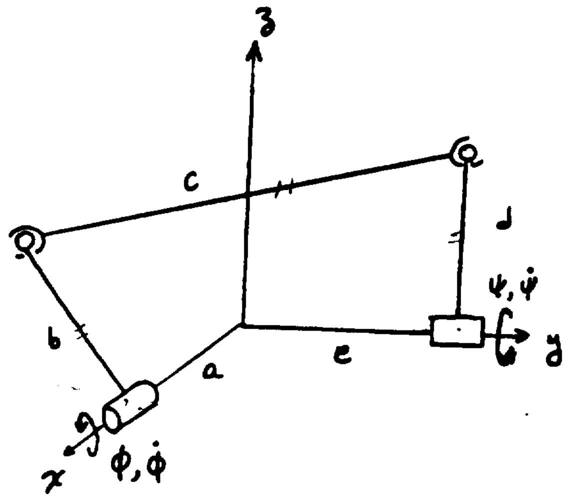
The system below represents a model for a device supported on a flexible foundation. The foundation is represented as a free mass.



1) Find the natural frequencies of this system.

2) Assume that a disturbing force $f(t)=P_0\sin(\omega t)$ is applied to mass m_1, and derive expressions for the motion of the foundation (m_2), and for the force transmitted to the foundation.

3) For vibration isolation, a general guide is that the natural frequency of the system on its support should be at most one-third the disturbance frequency. Suppose the device, m_1, weighs 10 times its foundation, m_2. For a fixed disturbance frequency ω_0, compare the required natural frequency for vibration isolation of this flexible foundation model to the required natural frequency if the foundation were rigid and fixed in space.



The figure shows a 4-bar spatial RSSR* mechanism. Two revolute joints rotate links b and d that connect to link c by ball and socket joints. Angles ψ and ϕ are assumed known and are measured from the x-y plane using a right-hand rotation. Given the link lengths a, b, ... e find:

a. The degrees of freedom of the linkage.

b. $\dot{\phi}$ as a function of $\dot{\psi}$.

c. The mechanical advantage for input ψ and output ϕ.

*RSSR - Revolute - Spherical - Spherical - Revolute

A rigid wheel has principal moments of inertial $I_1 = I_2 \neq I_3$ about its body-fixed principal axes $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$, as shown in Fig. 1.179. The wheel is attached at its center of mass to a bearing which allows frictionless rotation about one space-fixed axis. The wheel is "dynamically balanced", i.e. it can rotate at constant $\omega \neq 0$ and exert no torque on its bearing. What conditions must the components of ω satisfy? Sketch the permitted motion(s).