GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1996

DYNAMICS & VIBRATIONS
EXAM AREA

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Assigned Number **(DO NOT SIGN YOUR NAME)**

-- **Please sign your name on the back of this page** –

Instructions: **Closed book, closed notes**
Do any 3 of 4 problems
Clearly indicate which 3 problems you want to have graded!

Problem 1.

A thin homogeneous triangular plate of mass m, base a, and height 2a is welded to a light axle that can turn freely in bearings at A and B as shown in the figure below-left. It is known that the coordinates of the center of mass are $(x_C, y_C) = (a/3, 2a/3)$ and that the inertia matrix for the plate using the coordinate system xyz with origin at point A is given by:

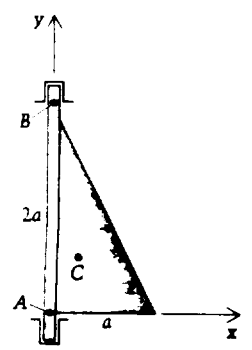
$$[I_A] = \frac{ma^2}{6}\begin{bmatrix} 4 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

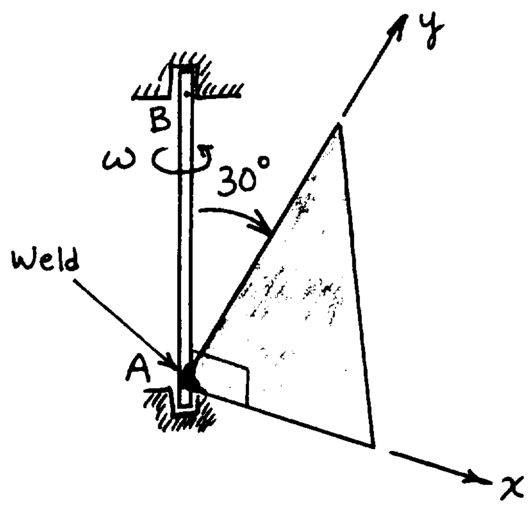
a) If the plate is turning at constant angular speed ω, find the torque that must be applied to the axle, and find the dynamic bearing reactions.

b) Find the principal axes at A and the principal moments of inertia there. Draw the axes on a sketch.

c) Find the work done in bringing the plate up to speed ω from rest.

d) If the plate is welded at point A to the shaft AB in such a way that the y-axis makes a 30° angle with the plate while the x-axis remains perpendicular (see sketch below-right), find the moment that must be applied to the plate at the weld location.



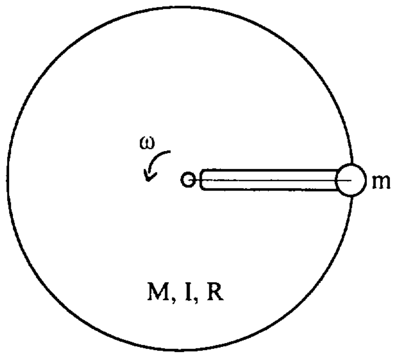


Problem 2.

A disk of mass M, radius R, and inertia I rotates freely about its center with angular speed ω. A point mass m travels in a narrow radial slot and is pulled towards the center by a light cable. At $t = t_0$, the mass is stationary at radial distance $r_0 = R$ and the disk has angular speed ω_0.

a) At $t = t_1$ the mass is brought to rest relative to the disk at distance $0 < r_1 < R$. What is the force F_1 in the cable?

b) At $t = t_2$ the cable is cut and then the mass exits the slot at time t_3. What is the exit speed of the mass relative to the disk?



Problem 3.

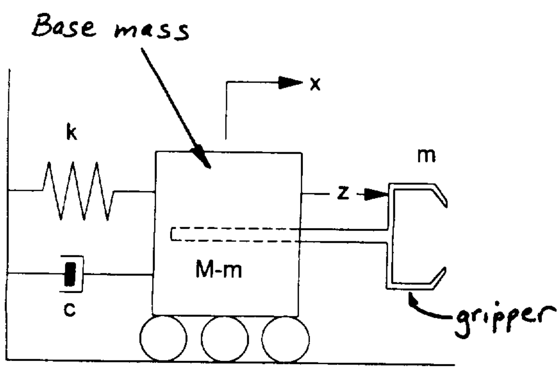
A servo-actuator can be represented as a base mass, (M-m), and a translating gripper, m, as shown. It is known that the gripper comprises 1/2 of the total system mass (m=M/2). The base mass is supported on a compliant mount with stiffness k and viscous damping c. By means of an internal motor, the displacement of the gripper relative to the base mass z(t) can be controlled to produce any commanded motion. When relative motion between the base and gripper was prevented (z(t)=0), it was determined that the system had a natural frequency of $\omega_n=2\pi$ rad/s and a damping ratio of $\zeta=0.1$.

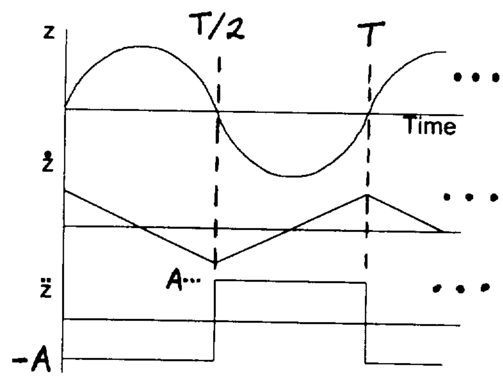
Consider the steady, periodic relative displacement profile displayed below along with the relative velocity and relative acceleration time histories. Note that the relative displacement is <u>not harmonic</u>. Instead, it is composed of parabolic arcs of alternating sign and duration T/2, each having constant acceleration magnitude. Thus, the relative acceleration is a square wave of amplitude A:

$$\ddot{z} = -\sum_{n=1,3,5,\ldots}^{\infty} \frac{4A}{n\pi}\sin(n\omega_0 t)$$

where ω_0 is the fundamental frequency of z(t), $2\pi/T$ rad/s.

(a) Find the differential equation of motion for the base mass displacement, x(t).
(b) Find the values of the period T that would be <u>most undesirable</u> in terms of the amplitude of displacement of the base mass.
(c) For the case where $\omega_0=\omega_n/3$, <u>estimate</u> the steady-state displacement response of the base mass, x(t). Discuss the nature of any approximation error incurred.
(d) If it is desired for the gripper to have an <u>absolute displacement</u> like the one specified in the figure, explain how one might obtain the necessary relative displacement that must be commanded in order to achieve the desired absolute gripper displacement.





Problem 4.

A stretched string of length L, mass per unit length ρ, and tension T, is fixed at one end ($x=0$) and is attached to a mass, m, which is constrained to move only in the y direction, at the other end ($x=L$). The mass of the string $m_s = \rho L$, is equal to the mass m, and hence **is not negligible.** The y displacement of the string is given by $w(x,t)$ which is assumed to be small.

a. Use a one degree of freedom model for the system to approximate the lowest natural frequency of the system. (3 points)

b. Use a two degree of freedom model for the system to approximate the two lowest natural frequencies of the system. Compare your result for the lowest frequency with that of part a. (4 points)

c. Considered as a distributed system, the string satisfies the equation of motion:

$$T\frac{\partial^2 w}{\partial x^2} - \rho\frac{\partial^2 w}{\partial t^2} = 0$$

(I) What are the boundary conditions for w ? (1 point)

(ii) What is the characteristic equation for the system? How many roots does it have? (2 points)