GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1997

Dynamics and Vibrations Ph.D. Qualifying Exam
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

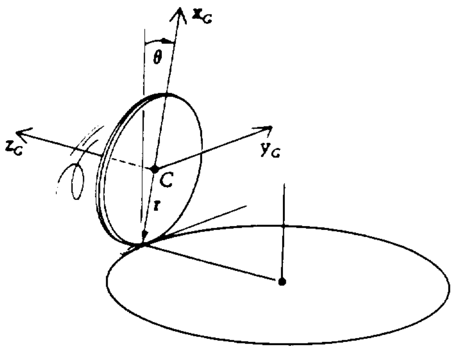
Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 1997

Instructions:

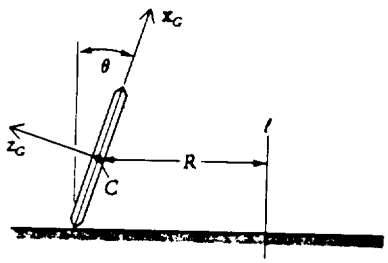
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end.

Problem 1.

Find the relationship among v_C, g, r, R, and θ such that the disk can roll without slipping around in a circle as shown in Figures (a) and (b), with v_C and θ remaining constant. Note that the mass moments of inertia of a thin disk of mass m and radius r is $mr^2/4$ about any diameter and $mr^2/2$ about the polar axis.



(a)



(b)

Problem 2.

You are returning from the ICoTUGD (International Conference on Teaching UnderGraduate Dynamics), after presenting the paper, "Do students understand 'free body diagrams' - engineering tool, political prisoner slogan, or anatomical giveaway?" While perusing the in-flight magazine you find the following advertisement:

LEVITATING [illegible]

[illegible] this precisely [illegible] balanced top will spin and float in mid-air, without electricity or strings to support it. Two magnets having the same polarity, one in the top and one in the base, oppose each other and force the top to float in air. This floating apparently defies a basic law of physics (Earnshaw's Theorem of 1848, which basically states that two magnets of like polarity cannot oppose each other without one of them turning over to its unlike polarity, thereby attracting it to the other magnet). Although this theorem remains true as long as there is no external force to prevent the magnets from attracting each other, the precise gyroscopic spinning motion of this top provides just enough force to keep the top floating without falling over. To use, simply hold the top above the base and spin. The top will remain levitated anywhere from two up to five minutes with practice. For easier spinning, we suggest the top starter sold below. Ages eight and up. Measures 1¼" H.

Levitating Anti-Gravity Top (described above). Floats 1¼ inches high.
649573 .... $39.95

Super-Levitating Anti-Gravity Top. As above, but levitates 2½ inches high.
774002 .... $49.95

Levitating Anti-Gravity Top Starter. For use with both models above, this electronic unit helps start the tops spinning. Plastic housing. Runs on two AA batteries (included). 6¼" L x 1¼" W x 1½" H. (3 oz.)
774011 .... $9.95

As a dedicated teaching assistant for ME 3760, you realize this would make an excellent class topic, even though it might be necessary to make some simplifying assumptions. While the students have had three-dimensional rotational dynamics, the topic of dynamic stability is somewhat too advanced for the class.

Your assignment is to provide a written handout solution to the dynamics homework question, "How does the top levitate while it's spinning yet fall when it's not." Make sure you use sentences, figures, variables, and equations. You may assume Earnshaw's theorem but clearly state all other assumptions.

Problem 3.

A robot of mass m_r is suspended from a thin pipe that vibrates in bending. We model it as a single lumped mass supported by a translational spring and damper.

a) Find good values of the model parameters to approximate the behavior of the physical system. Dynamic and static behavior should be considered. Express the parameters in terms of the physical parameters of the pipe that you clearly identify. If necessary in a practical sense, describe experiments that may be performed to determine the parameter values.

b) When the robot moves a payload of mass m_2 the pipe will deflect. Conversely, if the pipe is vibrating, movements of the pipe can be reduced by proper movement of the payload. If the motion of the payload is made to be emulate that of a mass m_2 on a spring k_2 and damper b_2, what are good values of k_2 and b_2 for the purpose of damping out vibrations? A schematic of the resulting translational model is shown below. You may assume $m_2 \ll m_1$ for this part of the problem.

c) Given that the actual system is more complicated than the simplified two-degree-of-freedom model shown, how would you determine if a translational model of the robot is suitable? Propose some quantitative criteria and how they can be evaluated analytically.

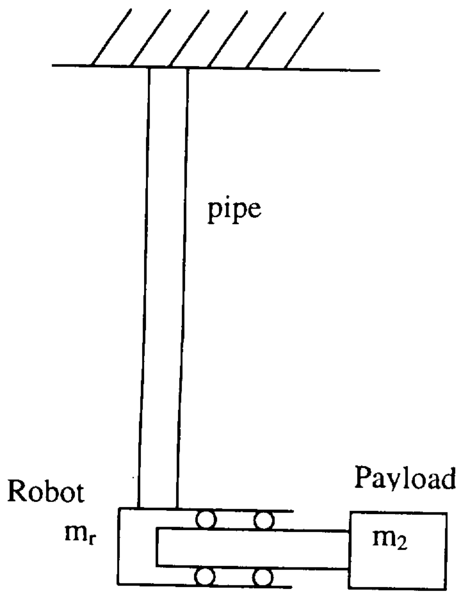
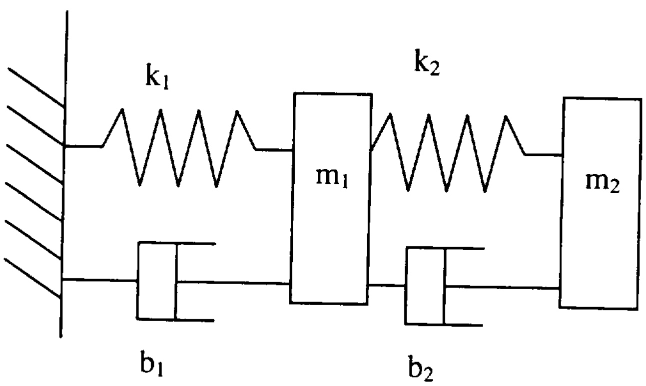


Diagram of physical system



Schematic of model for part b)

Problem 4.

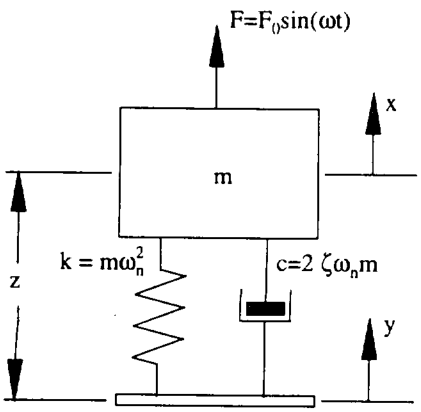
Two base isolation systems for identical masses are to be compared. System 1 consists of a linear spring in parallel with a viscous damper. System 2 has the same spring as System 1, but the viscous damper is replaced with an actuator force, f(t). This force is designed to emulate a viscous damper between the mass and the *inertial ground* sometimes termed a "skyhook damper." Thus the upward force on the mass is given by $f(t) = -c\dot{x}(t)$, where $\dot{x}(t)$ is the absolute velocity of the mass. In questions (a) and (b) assume that the applied force F(t) is zero.

(a) For the case of harmonic base excitation, $y(t)=Y\sin(\omega t)$, find the transfer function from y(t) to x(t) for System 1. Sketch the transfer function (frequency response function) X/Y versus ω for several values of damping ratio ζ. Be sure to note any features of interest such as the low frequency limit, high frequency limit, minima, maxima, etc.

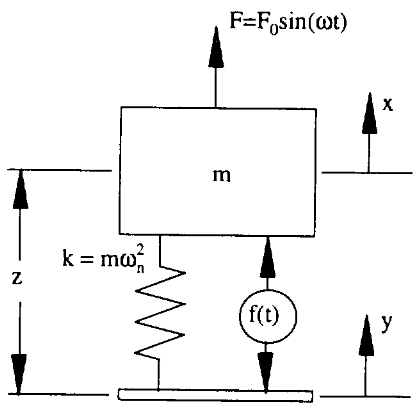
(b) Also for the case of harmonic base excitation, find the transfer function from y(t) to x(t) for System 2. Again, sketch the transfer function for several values of ζ and note any features of interest.

(c) If y=0 and $F(t)=F_0\sin(\omega t)$, find the transmissibility function for each System, F_T/F_0, where F_T is the magnitude of the force transmitted to the base.

(d) Based on your investigation in parts (a), (b), and (c), compare the two isolation system designs. What are the advantages and disadvantages of each design.



System 1



System 2