GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1998

Dynamics & Vibrations
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

Please <u>p r i n t</u> your name here.

The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 1998

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave any algebra until the end. Good Luck.

Problem 1

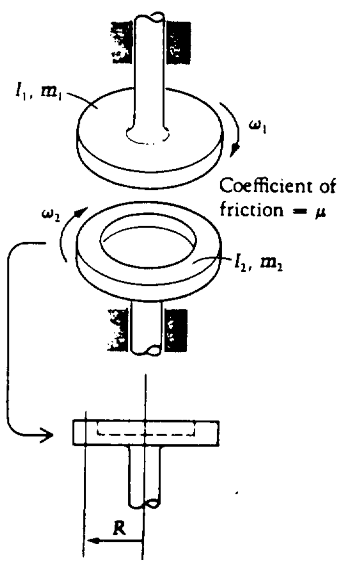
Two disks are initially spinning freely in the directions shown in the figure. The upper disk is made to descend very slowly until it is a very small distance above the lower disk. It then is released and falls into contact with the rim of the lower disk. (Assume that there is no rebound after the initial contact.)

1. Determine the common rotation rate the disks ultimately attain due to sliding friction at the rim.

2. Determine the elapsed time from the instant of release until the disks attain their common rotation rate.

3. Derive an expression for the energy loss. Show that if $I_1 = I_2$, then 100 percent of the initial mechanical energy is lost. Note: $\Delta E = KE_{initial} - KE_{final}$.

4. Identify which of the three answers (common angular speed, elapsed time, energy loss) would be the same if the two disks were locked together instantaneously, rather than slipping.



Problem 2

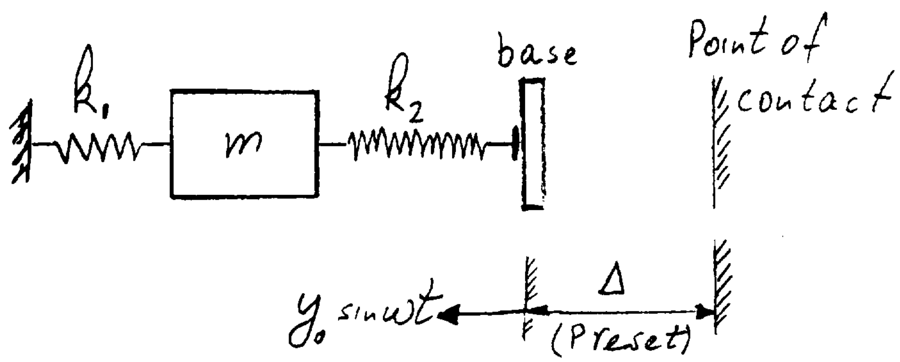
Consider the system described below in the figure. The two ends of spring k_1 are fixed to a rigid wall and to the mass, m. Left end of spring k_2 is fixed to mass, m, but its right end is NOT fixed to the base. The base is first brought into contact with the free end of spring k_2 at the "point of contact" (see figure). Then an additional displacement (preset), Δ, is forced upon the free end of k_2 from the "point of contact." About that new location the base is given an oscillatory motion, $y = y_o \sin(\omega t)$, where y_o is the amplitude and ω is the frequency of base oscillation.

A. Determine the condition upon the preset, Δ, which guarantees that the base and the free end of spring k_2 will NEVER separate (i.e., contact must be maintained at all time). -- (80%)

B. Between a "very stiff" or a "very soft" spring constant, k_2, which one would you choose as "best" for maintaining contact at all time. -- (20%)

Assumptions:

1. The mass of the springs is negligible compared to the mass, m.
2. The springs would never buckle under the compressions involved.

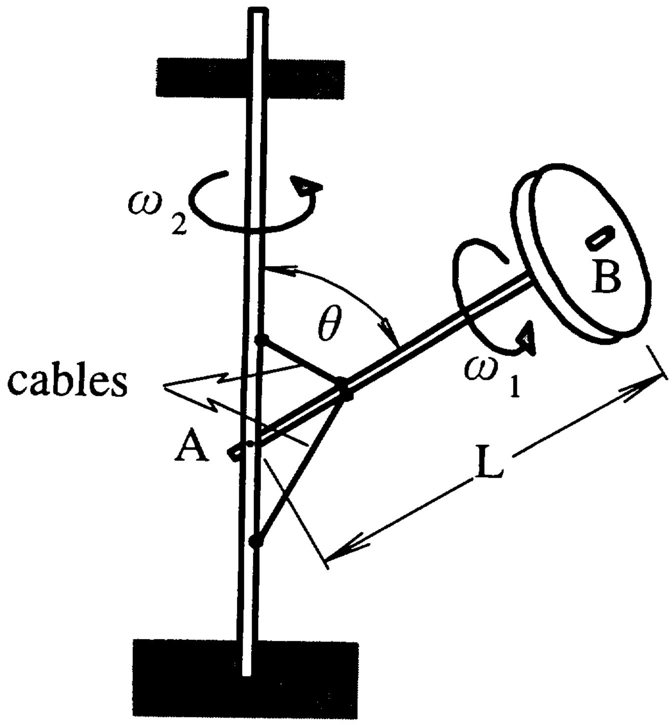


Problem 3

The flywheel spins freely at a very large angular speed ω_1 relative to shaft AB, counterclockwise as viewed looking from B to A. Shaft AB is pinned to the vertical shaft, and friction at joint A is negligible. The rotation rate of the vertical shaft is held constant at ω_2, counterclockwise when viewed downward. The flywheel's mass is m, and its principal moments of inertia about its centroid are I_1 about its axis of symmetry and I_2 perpendicular to its symmetry axis. The mass of shaft AB is negligible. In the initial motion, the angle θ is held constant by two cables, which are cut at $t = 0$.

1. At the instant immediately following the cutting of the cables, will θ tend to increase, decrease, or remain unchanged immediately after the cables are cut? Give a physical explanation, rather than mathematical analysis, to justify your answer.

2. Derive an expression for $\ddot{\theta}$ at the instant immediately after the cables are cut.



Problem 4

A rigid **massless** beam of length 2L connects masses m_1 and m_2 and rests on two identical springs of stiffness k. A third mass, m_3, is attached to the midpoint of the beam with a spring also having stiffness k as shown. For parts (a) and (b) below, assume that $m_1=m$, $m_2=2m$, and $m_3=m$. Please justify your answers fully; no credit will be given for guesses.

(a) Determine if one of the modes of this system is given by

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}.$$

(b) Determine if one of the modes of this system is given by

$$\begin{Bmatrix} y \\ \theta \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1/L \\ 2 \end{Bmatrix}.$$

(c) Now, consider the case of $m_1=m_2=m_3=1$ kg , L=1 meter, and k=1 N/m. Find the characteristic equation for this system and, if possible, find the natural frequencies and natural modes.