GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2000

Dynamics and Vibrations
EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2000

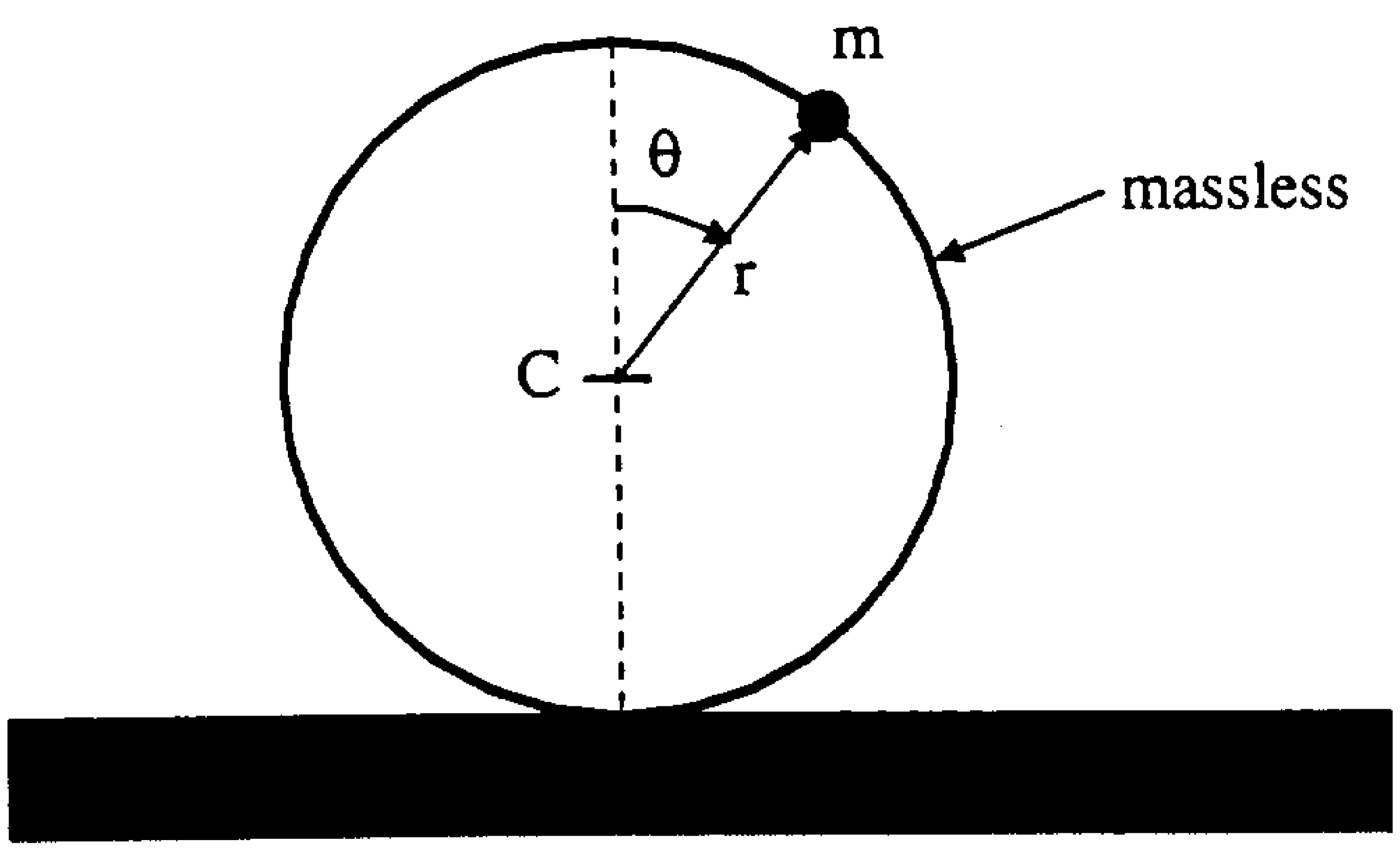
Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. Note: In order to ensure a more even playing field, **NO CALCULATORS ARE ALLOWED!** When necessary, you may leave your answers in terms of unevaluated numerical expressions.

Problem 1.

A point mass m is firmly attached to a *massless hoop* of radius r. The hoop is released from rest with the mass at the top, $\theta = 0$. Assume that there is sufficient friction to prevent slipping at all times.

(a) Give an expression for $\dot{\theta}$ as a function of θ which is valid for all time.

(b) Show that the hoop "hops" off the ground when $\theta = \pi/2$.



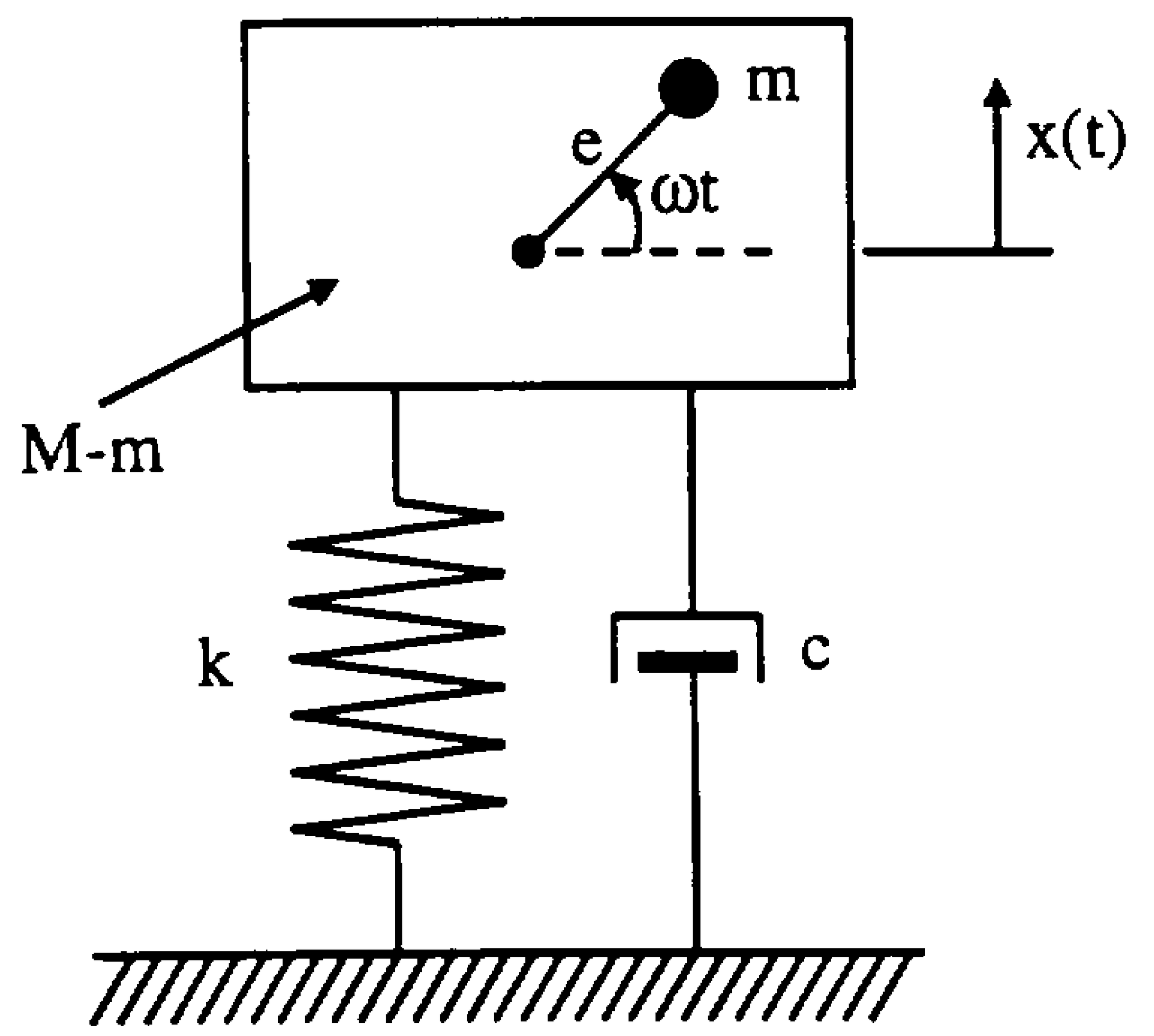
Problem 2.

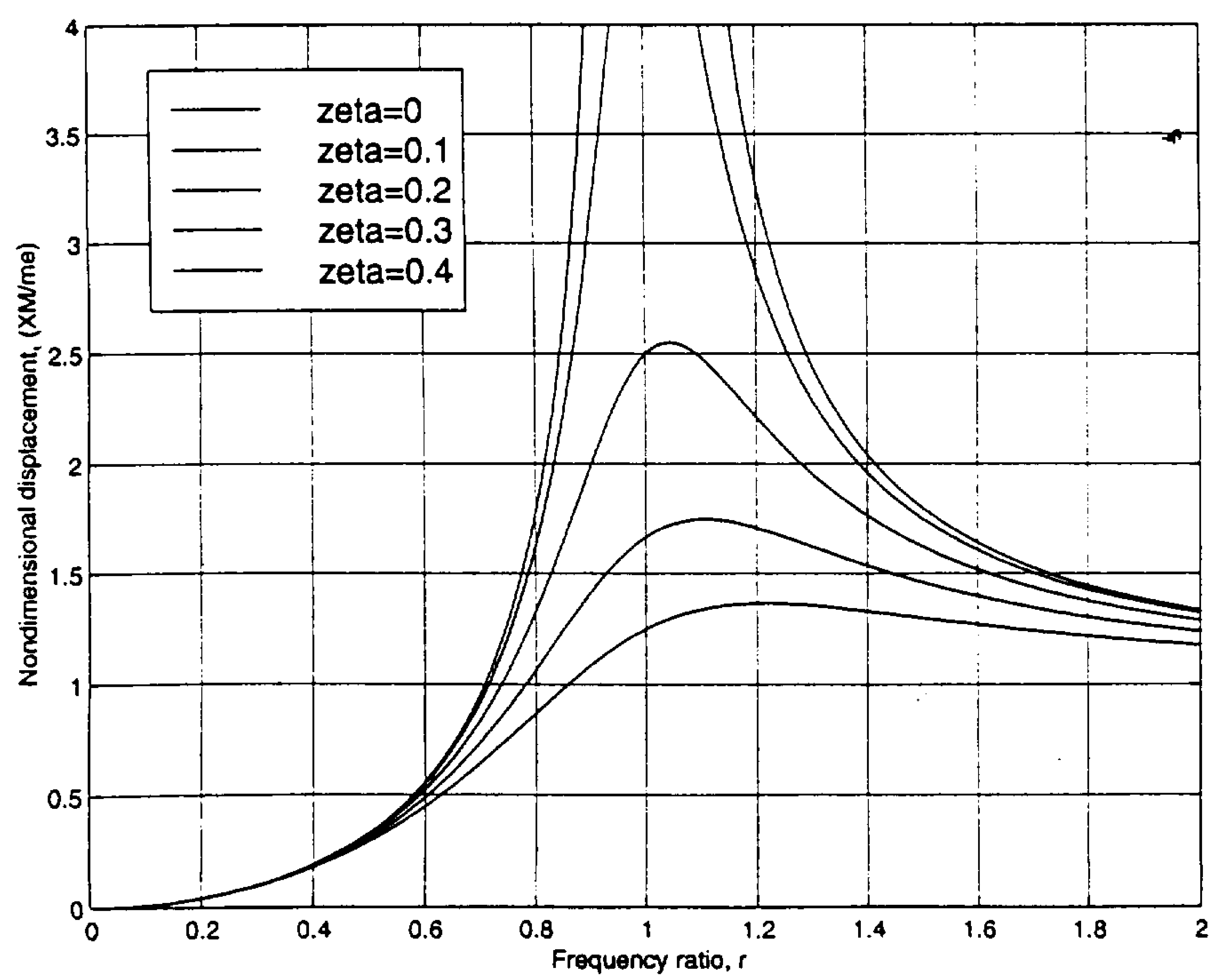
A rotating machine of total mass M=10kg, has a rotating unbalance with characteristics m=1kg and e=1cm. The machine is to rest on a resilient foundation which has yet to be selected. The operating range for steady-state operation is $0 \le \omega \le 200$ rad/s.

(a) Find the differential equation of motion for this system in terms of the machine displacement, $x(t)$. Note that $x(t)$ is the absolute displacement of the "machine housing" measured from the static equilibrium position.

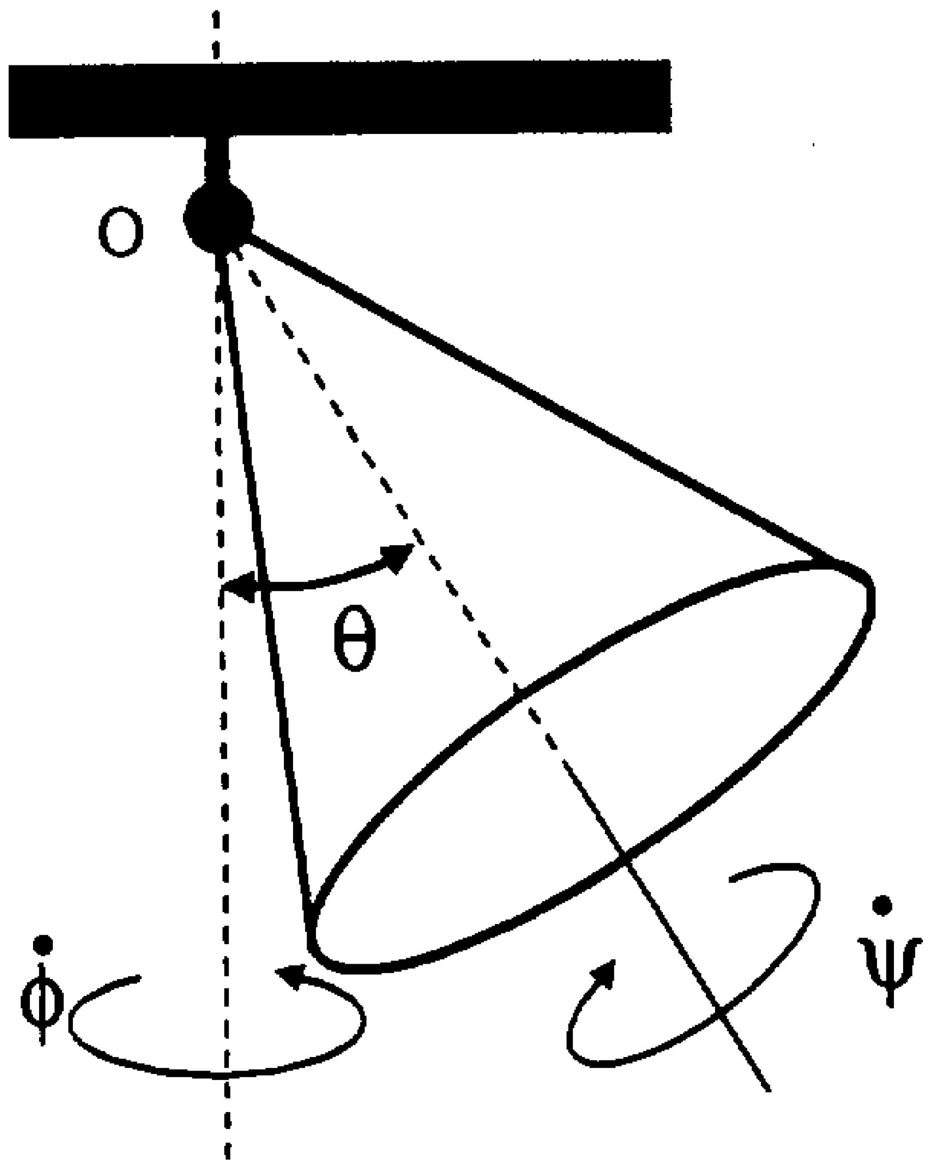
(b) For an undamped foundation, find all ranges of the spring stiffness k such that the amplitude of steady-state oscillation of the machine housing is no more than 25% of the unbalance length, e. You may use the graph below, which shows nondimensional displacement versus frequency ratio, $r = \omega/\omega_n$.

(c) Explain how damping influences your answer to part (b)? Be as specific as you can.

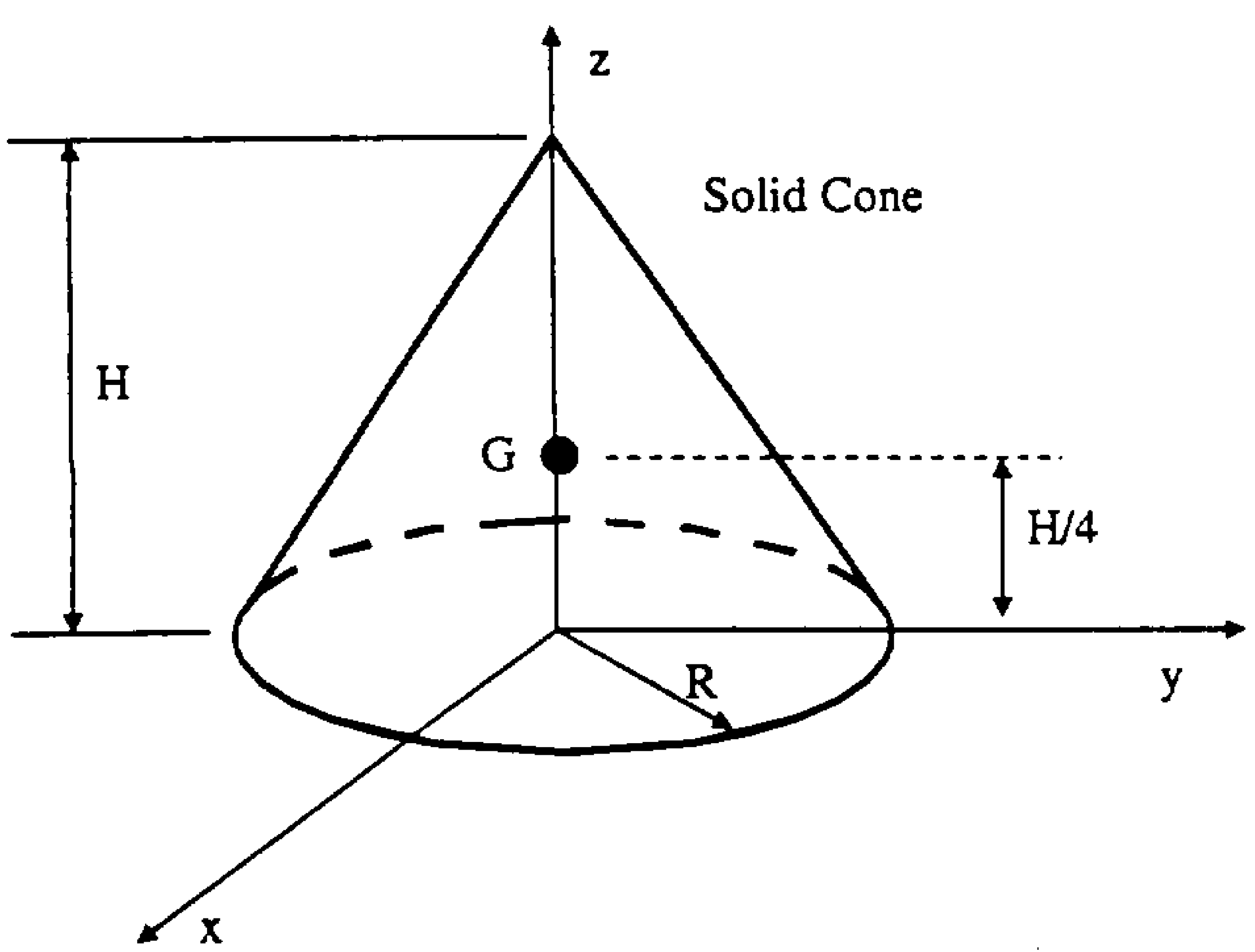




Problem 3.
A right circular solid cone of radius R and height H is supported at point O by a frictionless ball joint as shown. The inertia properties of such a cone are given below for coordinates given in the accompanying figure. If it is observed that the nutation angle θ is constant, give the relationship that must be satisfied between the spin rate, $\dot{\psi}$, and the precession rate, $\dot{\phi}$.



Inertia Properties:



$$I_{xx} = I_{yy} = \frac{m}{20}(3R^2 + 2H^2) \ ; \ I_{zz} = \frac{3}{10} mR^2$$

Problem 4.

The figure depicts the static equilibrium of a system that moves in the vertical plane. The horizontal force F acting on the small sphere varies harmonically at frequency Ω. The amplitude of this force is sufficiently small that the angle through which the bar swings is much less than 0.1 rad. The surface on which block m' slides is frictionless.

(a) For what combination of values of m' and k' does the bar remain stationary in steady-state corresponding to some specified Ω?

(b) When the conditions in part (a) apply, what is the amplitude of the vibration of the block?

(c) If $m' = m/2$ and $k' = 2mg/L$, what value(s) of Ω will cause the maximum vibration amplitudes for the block?