GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - FALL Semester 2001

Dynamics and Vibrations
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

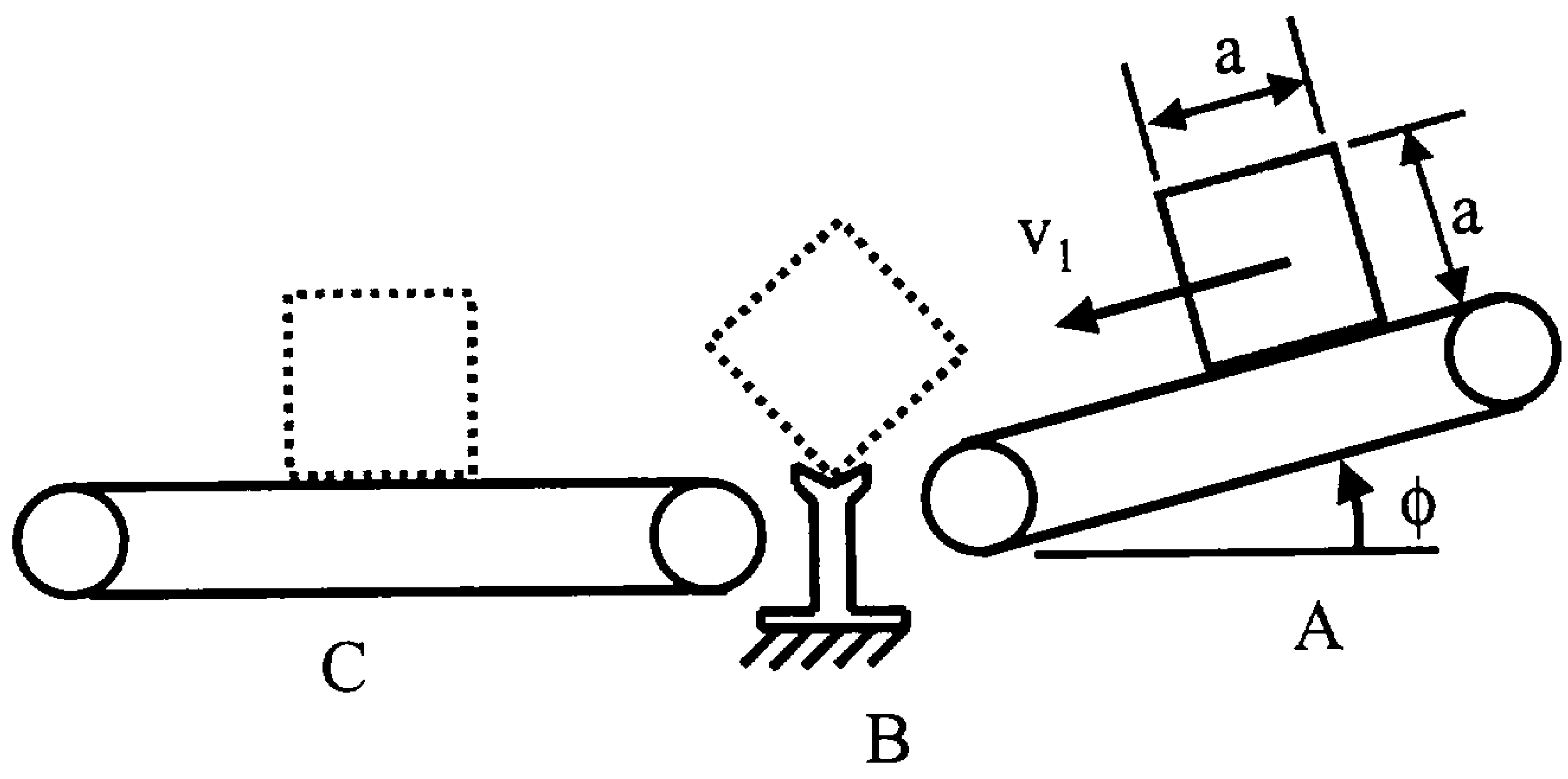
- **Please sign your name on the back of this page—**

Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2001

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

A square package of side a and mass m moves down an inclined conveyor belt *A* with a constant velocity $\vec{v}_1$. At the end of the conveyor belt, the corner of the package strikes a rigid support at *B*. Assuming that the impact at *B* is *perfectly plastic*, derive an expression for the smallest magnitude of the velocity $\vec{v}_1$ for which the package will rotate about *B* and reach conveyor belt *C*. The angle of conveyor *A* is $\phi = 15^\circ$ and the mass moment of inertia of a square plate about its center of mass is $I = ma^2/6$.



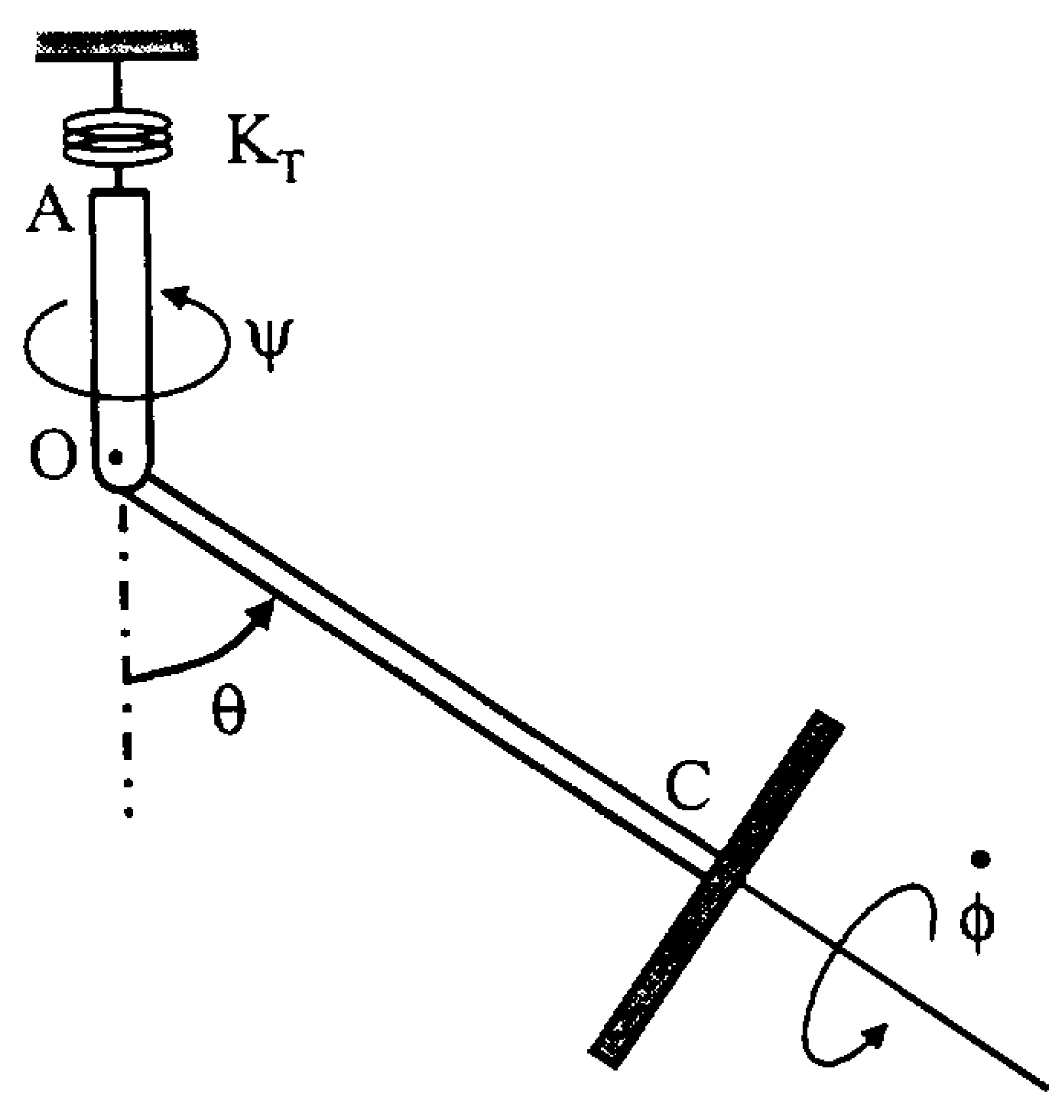
Problem 2.

A disk of mass m and radius R turns <u>freely</u> on shaft OC, with known initial rate $\dot{\phi}$. Shaft OC is massless, has length L, and is attached to the vertical member OA by means of a frictionless pin at point O. Finally, the vertical member OA is <u>massless</u> and is restrained by a stiff torsional spring K_T as shown. You may assume that K_T is large enough that the precession angle satisfies the relations $\dot{\psi} \approx 0$ and $\ddot{\psi} \approx 0$. Through a strain gauge on K_T, it is possible to measure the vertical torque, M, which must be applied to OA in order to prevent its rotation. Recall that the *polar* mass-moment of inertia of a thin disk is $I=mR^2/2$ and that the mass-moment of inertia of a thin disk about a diameter is $J=mR^2/4$. Answer the following questions, clearly defining any coordinate systems and/or notation that you introduce:

(a) Find $\ddot{\phi}$.

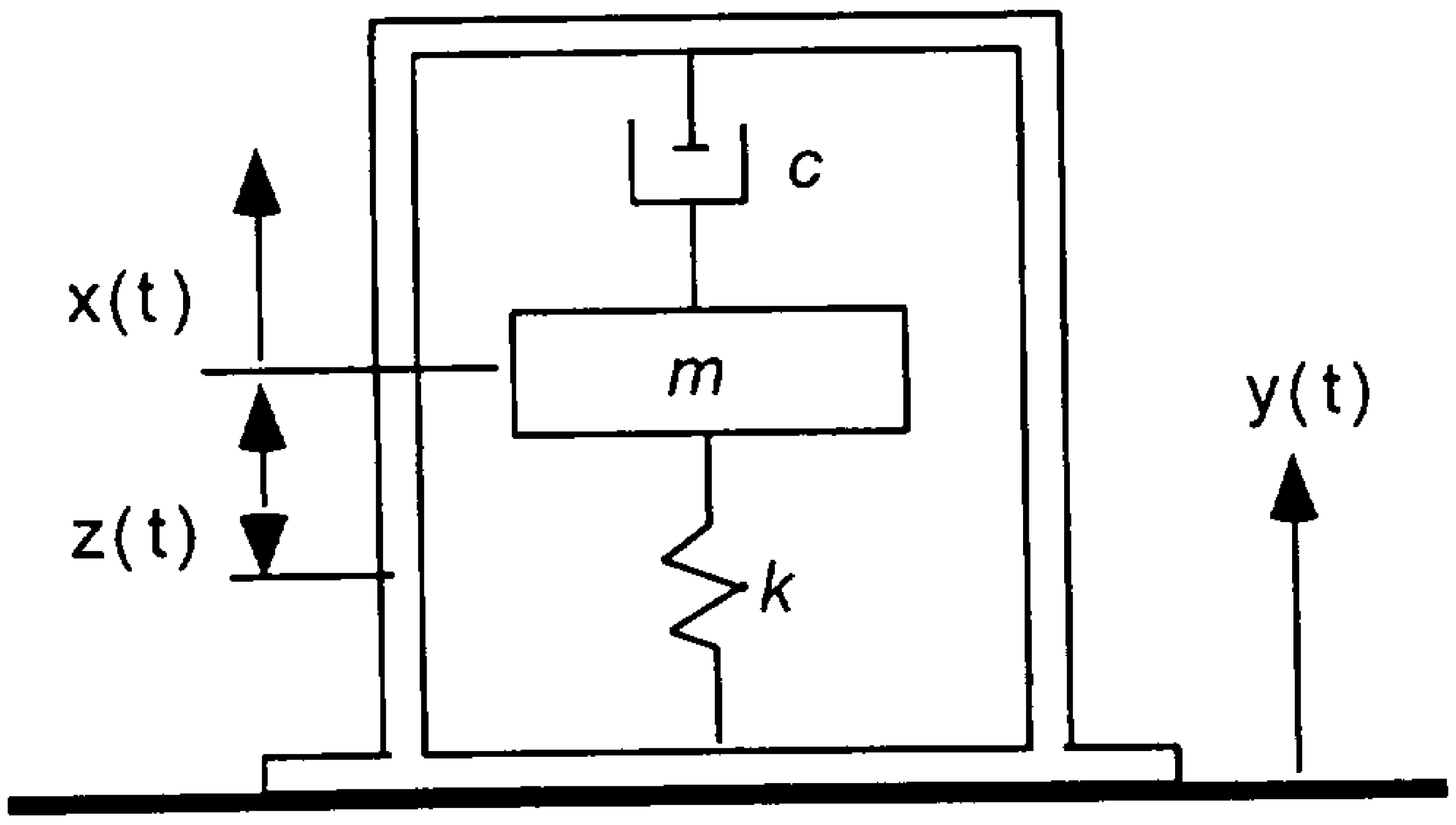
(b) Find a differential equation for $\theta(t)$.

(c) Find an expression for the torque M that must be applied to OA. Explain how one could use the measured value of M, together with a measurement of the nutation angle θ, as a way of obtaining the instantaneous value of the nutation rate, $\dot{\theta}$.



Problem 3.

Consider the spring-mass-damper system depicted below. The displacement of the mass is x, while the displacement of the support or base is y. The relative displacement between the mass and its housing is represented by z. If the base is subjected to a harmonic displacement, $y(t)=Y_o\sin(\omega t)$, derive the conditions for which measurements of the relative displacement are approximately proportional to the acceleration of the base. Also, derive the conditions for which measurements of the relative displacement are approximately proportional to the displacement of the base. In each case, give an expression for the deviation from exact proportionality. Clearly state all of your assumptions, approximations, etc.



Problem 4

A two-degree-of-freedom system is subjected to a harmonic force $F = 500\sin(\omega t)$ N acting at displacement point 1. The inertia matrix for the system is

$$[M] = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix} \text{ kg}$$

Damping is very light, and proportional to the stiffness matrix. The resulting steady state response has the form $z_n = \text{Re}\{Y_n \exp(i\omega t)\}$. Measurement of the response over a range of frequencies yeilds the plot of the magnitude of each $|Y_n|$ provided below.

(1) Express the solution for each $Y_n(\omega)$ in terms of a sum of contributions from each vibration mode.
(2) Based on an examination of the plot, what are the natural frequencies of this system?
(3) Based on an examination of the plot, what is the ratio of the mode coefficient magnitudes $|\phi_{2k}/\phi_{1k}|$ for each mode, $k = 1$ and $k = 2$?
(4) A normal mode $\{\Phi_k\}$ is defined such that $\{\Phi_k\}^{\text{T}}[M]\{\Phi_k\} = 1$. Based on an assumption that $\phi_{21}/\phi_{11} > 0$ and $\phi_{22}/\phi_{12} < 0$, what are the corresponding normal mode vectors for this system?
(5) Certain special characteristics of these plots indicate the fundamental nature of the spring-mass model representing this system. Draw a sketch of this model, and justify your answer.