GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff

School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2004

DYNAMICS & VIBRATIONS

EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

✶ Please sign your **name** on the back of this page —

## Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2004

Instructions:

Please work 3 of the 4 problems on this exam. <u>It is important that you clearly mark which three problems you wish to have graded</u>. For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

The homogeneous rectangular block of mass m is centrally mounted on the shaft A-A about which it rotates with a constant speed $\dot{\phi} = p$. Meanwhile, the yoke is forced to rotate about the x-axis with the constant speed ω_0. Find the magnitude of the torque $\mathbf{M}$ as a function of ϕ. The center O of the block is the origin of the x-y-z coordinates. Principal axes 1-2-3 are attached to the block. Note that:

$$I_1 = \frac{m}{12}\left(a^2 + b^2\right), \quad I_2 = \frac{m}{12}\left(b^2 + c^2\right), \quad I_3 = \frac{m}{12}\left(a^2 + c^2\right)$$
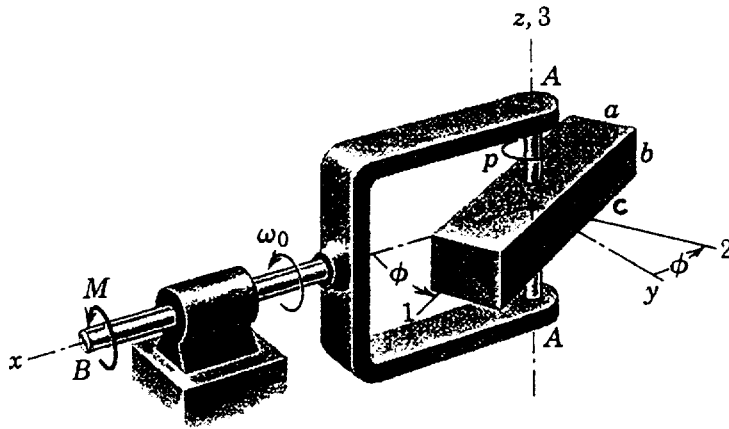
Problem 2.

A pendulum of length L and point-mass m hangs beneath a cart as shown. The cart has mass M and is restrained by two springs having stiffnesses k_1 and k_2. In addition, the cart is excited by means of a horizontal force $F(t) = F_0 \sin(\omega t)$.

(a) Find the equations of motion for this system.

(b) For the case of $k_1 = k_2 = \dfrac{mg}{L}$, $L = g$ (in magnitude), and $M = 2m$, find the natural frequencies and natural modes of the linearized system.

(c) Plot the mode shapes

(d) Find the generalized masses and stiffnesses

(e) Decouple the equations of motion using the modal matrix

(f) Solve for the steady-state motion of the system.
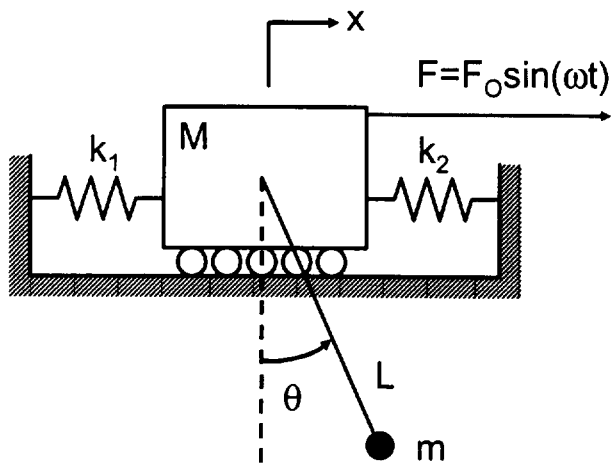
Problem 3.

A uniform disk of mass m and radius R is attached to a thin, massless rod AB having length L. The rod is attached to a massless slider by means of a frictionless pin at point B. The block slides without friction against a horizontal surface and does not lose contact with the surface at any time. Initially, the rod is vertical ($\theta = 0$) with zero velocity and is given a very small disturbance. In the questions below, you are to compare/contrast two different cases: 1. the case where the disk is attached to the rod by means of a frictionless pin at point A, and 2. the case where the disk is "clamped" to the rod such that no relative rotation between the disk and the rod takes place. The mass moment of inertia of a uniform disk about its polar axis is $I_A = mR^2/2$.

(a) In case 1, find the velocity of point A just before impact.

(b) In case 2, find the velocity of point A just before impact.

(c) For each case, calculate the acceleration of point A when the rod has fallen to an angle of θ ($0 < \theta < \pi/2$). Your answer should be expressed as a (vector) function of θ.

(d) Discuss how you would find the time of impact in each case.
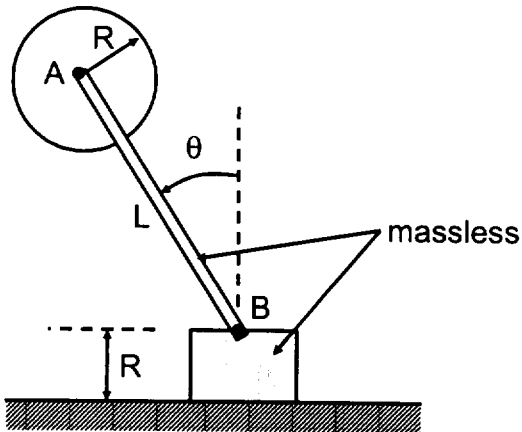
Problem 4.

The generalized mass of a one degree of freedom system is 500 kg. The system is subjected to a harmonically varying force and the response is measured. In the below graph the response is the solid line $x(t)$, and the force is the dotted line $Q(t)$. The horizontal axis is time, measured in seconds. Each quantity is plotted as a fraction of its maximum value. The measured values are $\max(x) = 1.6667$ mm, and $\max(Q) = 10$ kN. Based on this information and the graph:

(a) What is the actual function $Q(t)$?

(b) What is the system's natural frequency and critical damping ratio?

Be careful to fully explain all calculations.