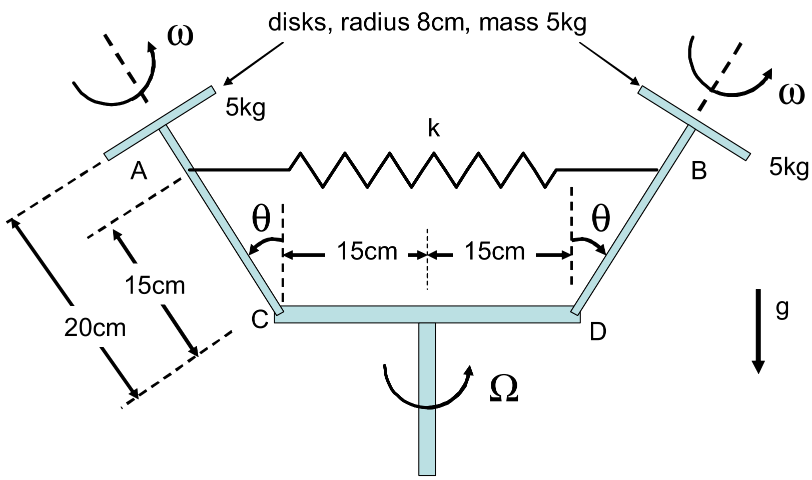
Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2005

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

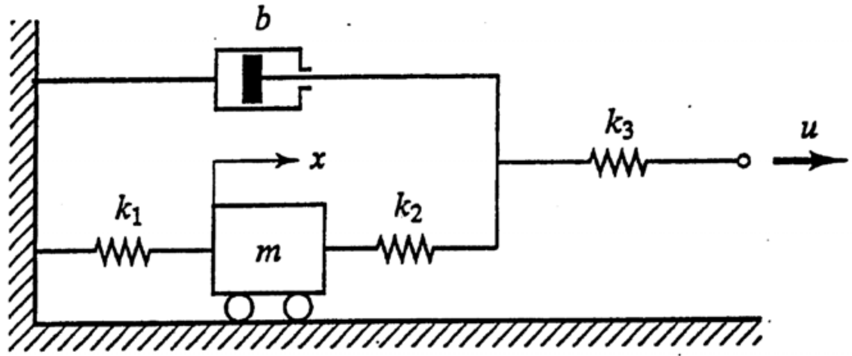
Problem 1.

The symmetric mechanism shown in the figure rotates about the vertical axis at a constant angular speed of $\Omega = 3$ rad/s. The vertical shaft is rigidly fixed to a horizontal link *CD*, which is pinned at one end to link *AC* and at the other end to link *BD*. Two identical disks of radius $R = 8$ cm and mass $m = 5$ kg are mounted at points *A* and *B* on the ends of links *AC* and *BD*, respectively, and spin with equal rates ω about the axis of the respective link. A spring with an unstretched length 30 cm and spring constant $k = 320$ N/m connects link *AC* to link *BD* at the illustrated points. Determine the angular speed ω, and its direction, which is required to maintain an angle $\theta = 30^{O}$ between the vertical axis and links *AC* and *BD*. The mass of the shafts and spring is negligible.



Problem 2.

Consider the mechanical system shown in the figure. The system is at rest for t<0. The input displacement, u, is given at t=0. The displacement of the mass, x, is measured from the equilibrium position. Obtain the transfer function for input/output (X/U), and determine the response for the case that u is sinusoidal, assuming all parameters equal to one unit in consistent SI.

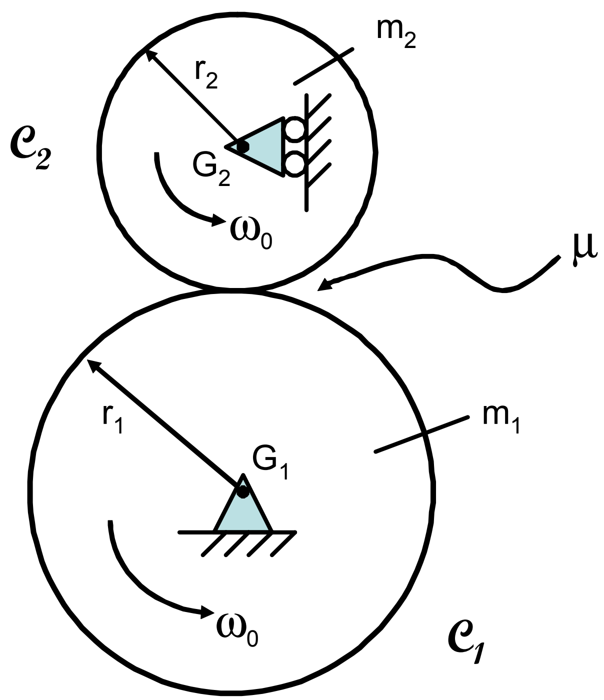


Problem 3.

Consider two homogeneous cylinders or disks C_1 and C_2, of masses m_1 and m_2 and radii r_1 and r_2 respectively. The moments of inertia about their corresponding geometrical and mass centers G_1 and G_2 (parallel to their generators), $I_1 = m_1 r_1^2/2$ and $I_2 = m_2 r_2^2/2$. C_1 can rotate freely about a fixed axis through G_1, parallel to its generators and perpendicular to a vertical plane; C_2 can rotate freely about a similar axis through G_2 parallel to its generators and to the first axis (i.e., both axes are horizontal and mutually parallel). As indicated in the figure however, the upper cylinder can slide freely in the vertical direction through G_2. Initially C_1 and C_2 are rotating, both with the same angular velocity ω_0 (*same magnitude and sense for both cylinders*), and then they are brought into contact.

(a) If the friction coefficient at the contact point is μ, how long will it take for C_1 and C_2 to roll on each other, and what will their angular velocities be then? Express your answer $t_{rolling} = t_R$ in terms of ω_0, r_1, r_2, μ, g (gravity), m_1, and m_2.

(b) What happens if $m_1/m_2 = r_1/r_2$?



Problem 4.

Shown below is a highly simplified model of a bladed rotor disk for jet engine applications. In general, each blade has a mass m, stiffness k_1 to the rotor disk (assumed stationary) and has blade-to-blade stiffness k_2. Because of the way the "first and last" blades are connected, such systems are said to be *cyclic*.

(a) Assuming very small displacements, give the equations of motion for a 3-blade version of this type of system.

(b) Show (verify) that the eigenvectors of the 3DOF system are given by

$$\phi_1 = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}, \quad \phi_2 = \begin{Bmatrix} 1 \\ \cos(2\pi/3) \\ \cos(4\pi/3) \end{Bmatrix}, \quad \phi_3 = \begin{Bmatrix} 0 \\ \sin(2\pi/3) \\ \sin(4\pi/3) \end{Bmatrix}$$

(c) Find the natural frequencies of the system.

(d) A very common type of forcing for turbomachinery rotors is *phased harmonic excitation* of frequency ω. For the nth blade in an N-blade system, the excitation force has the complex form:

$$F_n(t) = \mathrm{Re}\{F \exp(i(n-1)2\pi/N)\exp(i\omega t)\}$$

Note that each blade sees the same excitation, delayed by an amount corresponding to its position on the rotor. For the 3DOF system, find the equation that would yield the complex amplitudes of response; solve it only if time permits.