GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Semester 2007

DYNAMICS & VIBRATIONS

EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

✱ Please sign your **name** on the back of this page —

PROBLEM 1

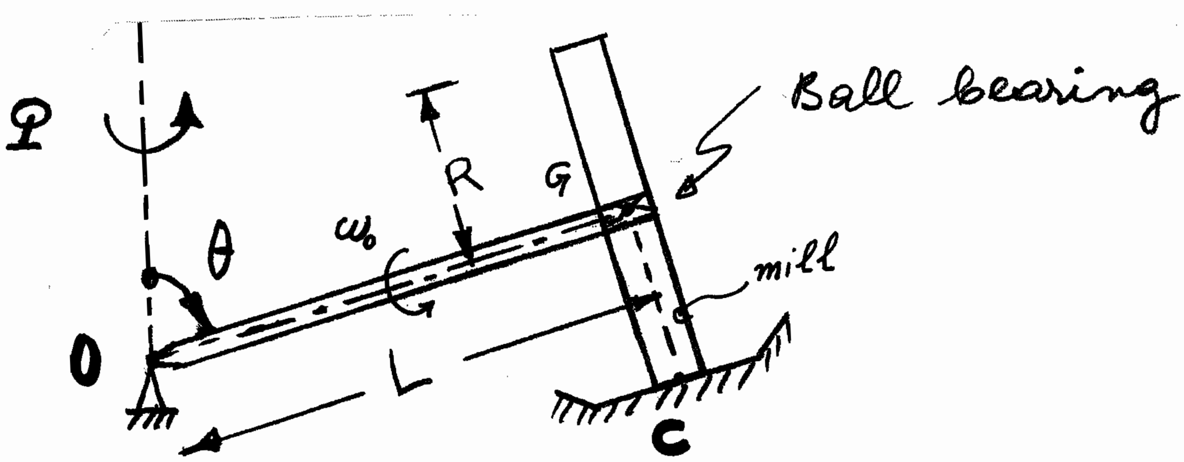
A homogeneous grinding mill of weight W and radius R can turn freely about its axle (i.e. smooth ball bearing at G, $OG = L$ — see Figure). The axle is assumed weightless, and is attached to a smooth ball-and-socket joint at the fixed point O, and the assembly is forced to precess at a *constant* angular velocity Ω about the vertical axis through O. The moments of inertia of the mill about an axis parallel to its diameter, through the mill's geometrical center and mass center G, is $A = mR^2/4$, while that about a "polar" axis through G is $B = mR^2/2$. Find:

a) The rate of spinning (i.e. angular velocity) of the mill, say ω_o, about the axle in terms of Ω, R, L, θ.

b) The (inertial) velocity and acceleration vectors (i.e. components relative to convenient axes) of G, in terms of L, Ω, θ.

c) The normal forces (vector → components) between the inclined track and the mill, i.e. at the (practically smooth) contact point C, say from the track to the mill, in terms of W, m, Ω, θ, L.

d) The reaction force (vector → components) at the pivot O, again from O to the axle, in terms of W, m, Ω, θ, L.



Problem 2

A massless wheel of radius R=*0.5m* is suspended at its center (point *O*) by a spring, $k = 10^5\ N/m$, from the ceiling. A massless chain, which is wrapped around the said wheel (similar to wheel-and-chain of a bicycle), supports a heavy hanging mass, *m=100 kg* (see Figure). The system is allowed to deflect first quasi-statically, reaching static equilibrium. From that position and with zero initial conditions, a force $F = 10\sin(5\pi t)$ is applied to the wheel at point *P*, which is located at *R/2* from point *O* (as shown). Do the following:

1. Write the governing equations for the system motion (by any technique of your choice).
2. Determine the static deflection of the system.
3. Determine the total dynamic response of the mass (excluding the static deflection), and analyze whether the system response becomes unbounded for the parameters given. State whether the system response is in-phase or out-of-phase with respect to the force.
4. A viscous damper is now added in parallel to the spring. Determine the damping coefficient for the damping ratio to become 0.1.
5. Including the damper, determine the maximum steady-state dynamic response of the mass and the phase angle.
6. Suppose that the force, *F*, can be transferred to any other point on the wheel. Is there such a point for which the system dynamic response would be zero? If yes, identify the point, and if no, explain why not.

Advice:

Writing correct EOM is paramount! Wrong equation(s) shall have a detrimental effect on the entire problem. So make sure that you have the correct EOM.