Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2008

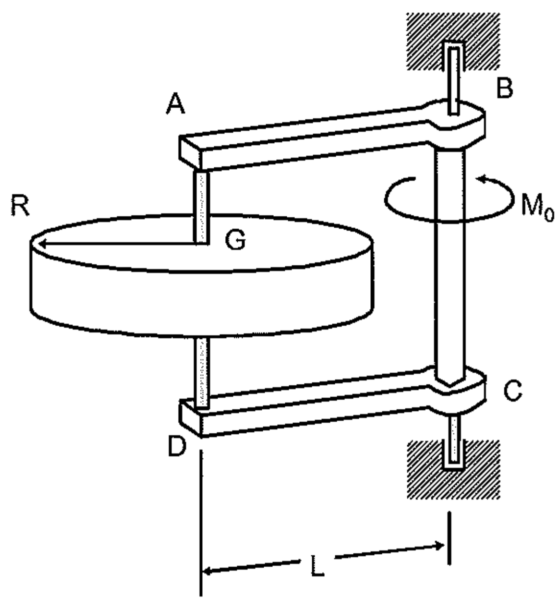
Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

A rigid cylindrical rotor has a mass, m, and a radius, R. The disk is mounted on an axle A-D of a rigid, massless, outer gimbal, $ABCD$; axle A-D passes through the center of mass of the disk, G. The bearing at location A is faulty, and there is a lot of Coulomb friction at that location. The disk is initially at rest and bearing A is stuck relative to the outer gimbal when a torque of magnitude M_0 is applied to the outer gimbal about the vertical axis B-C. The mass moment of inertia of the disk about its mass center is $I_C = mR^2/2$. The maximum torque through the bearings before slipping occurs is M_f, but after slipping occurs, it takes a torque of only $M_s < M_f$ to keep it slipping.

(a) Find the maximum torque, M_0, that can be applied to the outer gimbal before slipping occurs at the bearing A.

(b) If the constant moment M_0 exceeds the level found in part (a), how many rotations will the disk experience every time the gimbal turns through 360°.

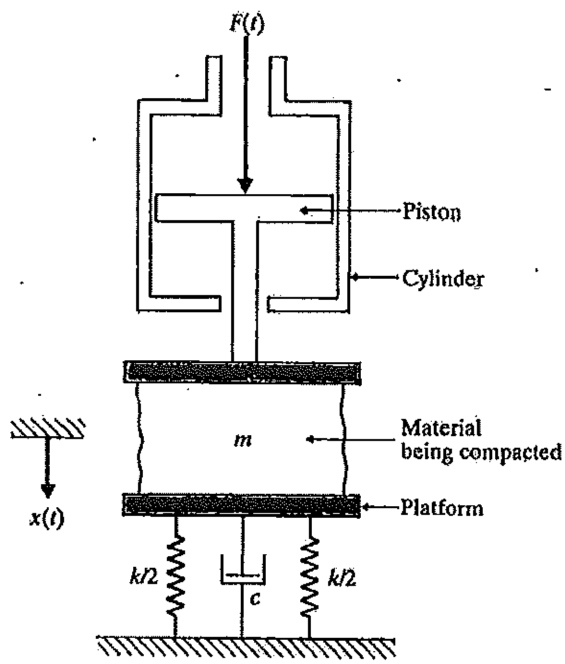


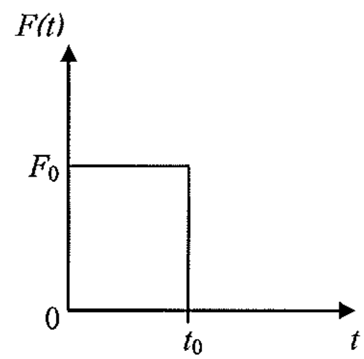
Problem 2.

A compacting machine, as modeled in the left figure, is to be treated using a single degree of freedom. The force ultimately acting on the mass due to a sudden application of pressure can be idealized as a step force, as shown in the right figure. Note that the mass m includes the mass of the piston, the material being compacted, and the platform. The compacting occurs over several loading cycles, however the damping is such that all oscillations decay before the next loading cycle commences.

a) List the assumptions necessary to model the system using a single degree of freedom. Among other considerations, address the stiffness of the compacted material versus that of the platform's support.

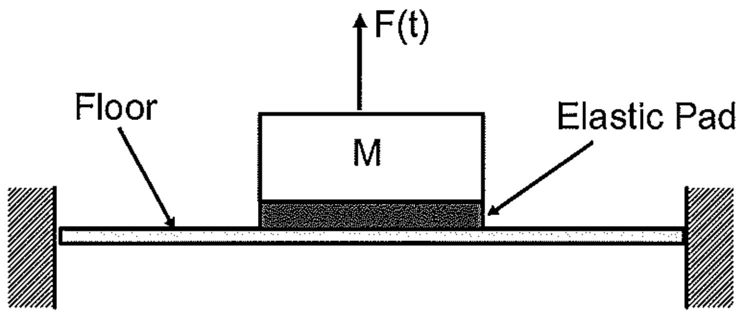
b) Determine the system's response as a function of time, during and after the loading cycle. **Be sure to express your answer using the provided parameters (m, k, c, t_0, F_0), or using quantities (e.g., natural frequency) for which you have provided expressions related to the provided parameters.**





Problem 3.

A transformer of mass M=1000 kg is installed on an elastic pad resting on a floor. The pad has been designed to statically deflect 1 mm under the weight of the device. The transformer generates vertical reaction loads, which may be assumed to act at its center of mass, described by $F(t) = F_0 \sin(2\pi ft)$ where f is 120 Hz.



1) What is the design force transmissibility ratio?

2) In operation, it is found that there is very little dynamic deflection across the pad, as the floor and transformer have approximately the same amplitude of vibration. Propose and justify a model for the system that explains this behavior.

3) Use your model from part 2 to determine the forced-response for the floor (symbolically), and use that forced response prediction to determine what must be done to the support to minimize the motion of the floor (adding mass to the transformer or floor is not an option, nor is modifying the floor). You may neglect damping in your discussion.

Problem 4.

A thin, uniform, square plate of side length L and mass m is welded at corner B a constant angle θ on shaft AB, while shaft AB rotates with variable rate $\dot{\psi}$. The centroidal inertia properties of the square plate are $mL^2/12$ about the in-plane principal axes and equal to $mL^2/6$ about the normal axis.

(a) (4 points) Ignoring gravity, find the total moment that must be applied to the plate at corner B to maintain this motion.

(b) (4 points) Give an expression for the kinetic energy of the plate. You do not need to carry-out any cross-products or matrix multiplications, but define all of your terms.

(c) (2 points) Consider the situation where the weld at location B is replaced by a pin such that the plate can rotate freely with angle θ relative to shaft AB, and AB is driven with constant rate $\dot{\psi} = \Omega$. Ignoring gravity, it can be shown that the plate can undergo small, stable oscillations about $\theta = 45^{\circ}$. Explain a procedure that you would use to predict the natural frequency of this oscillation.