

Dynamics and Vibrations Qualifying Examination

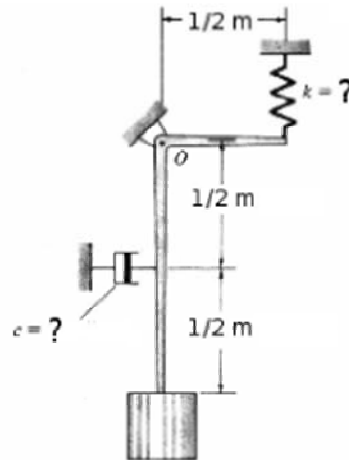
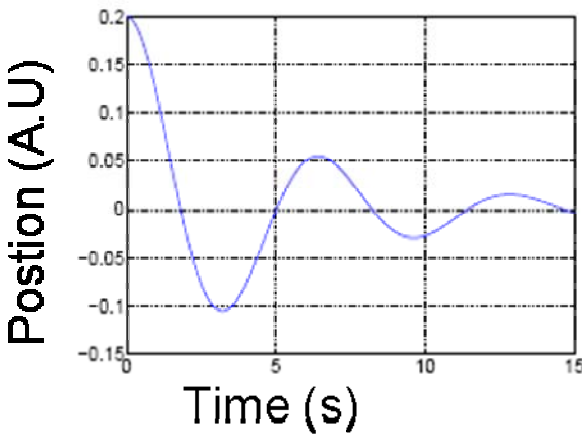
Fall 2009

Instructions: Work 3 out of 4 problems. If you submit work for all 4 problems, only the first 3 will be graded.

There is an equation sheet at the back of the exam.

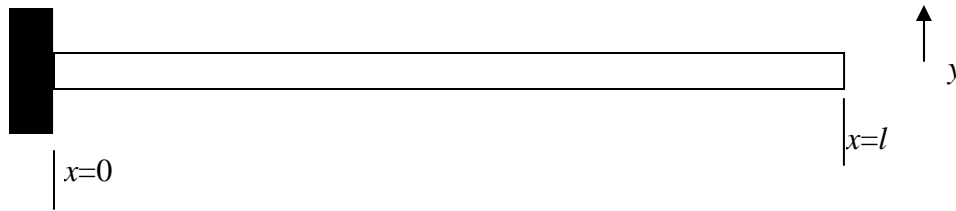
1) The bell-crank mechanism consists of a bent rod and an attached 5-kg weight. The rod has negligible weight, and the mass has negligible size. The weight is deflected slightly and released. The ensuing angular oscillations are monitored, and a shaft encoder provides the position data shown in the diagram where the time scale (in second) is calibrated but the position scale is not (meaning that we do not know the value of a unit).

Note: Only part (e) requires the answer to part (a). Parts (b), (c) and (d) can be done without (a). The grading for part (e) will be based on your answer in part (a), so as not to mark down twice for errors in part (a).



- Using Newton's laws, find the equation of motion.
- Find the damped natural frequency, ω_d .
- Find the damping factor, ξ_c .
- Find the undamped natural frequency, ω_n .
- Find the spring constant, k and damping coefficient, c

2) Consider transverse vibration of the cantilever beam depicted in the figure. The beam has length l , uniform section properties EI , and mass per unit length ρ .



a) What are the boundary conditions at $x=0$ and $x=l$?

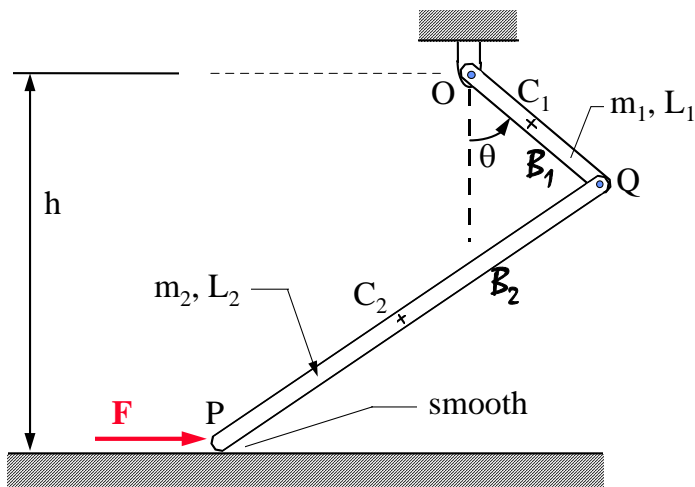
b) Use Rayleigh's method with an approximate solution $y = \left(\frac{A_0 l^2 x^2}{4} - \frac{A_0 l x^3}{6} + \frac{A_0 x^4}{24} \right) \sin \omega t$

(which is based on the static deflection of the beam under uniform transverse loading) to estimate the fundamental natural frequency of the beam.

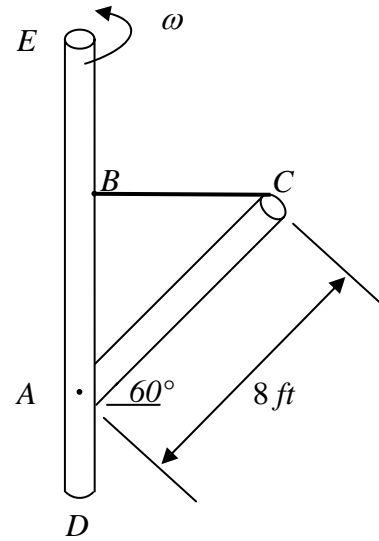
c) If one uses an approximate solution $y = A_0 x^2 \sin \omega t$ and Rayleigh's method, the resulting estimate of the fundamental natural frequency is significantly higher than the one obtained in part b; which estimate of the natural frequency is more accurate, and why? Speak both to the nature of solutions obtained with Rayleigh's method and to the trial functions themselves.

d) How would your analysis change if there was a point mass located at the tip of the beam? Would you expect the resulting estimate of the fundamental natural frequency to be higher or lower than those obtained in parts b and c?

- 3) A uniform link \mathcal{B}_1 of mass $m_1 = 2$ kg and length $L_1 = 1$ m is supported at location O by means of a frictionless pin. Another frictionless pin at end Q connects \mathcal{B}_1 to the uniform link \mathcal{B}_2 which has mass $m_2 = 5$ kg and length $L_2 = 5$ m. End P of \mathcal{B}_2 rests on a smooth horizontal surface while a constant force $F = 50$ N is applied to the right. The vertical separation distance between point O and the horizontal surface is $h = 4$ m. The two-link system starts at rest with $\theta = 0$ (\mathcal{B}_1 vertical).
- (a) Find the angular velocities of links \mathcal{B}_1 and \mathcal{B}_2 when $\theta = 90^\circ$ (\mathcal{B}_1 horizontal). Concentrate on setting up the correct relations and describing the process by which you would get the answer. Be as specific as you can.
- (b) Find the initial angular accelerations of the two links (magnitude and direction) just after the force is applied ($\theta = 0$). Again, describe the process by means of words, equations, and diagrams.

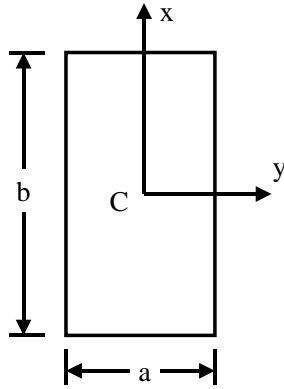


4) The slender rod AC of length 8 ft and weight 40 lbs. is pinned at A to a vertical axle DE which rotates with a constant angular velocity ω of 15 rad/sec. The rod is maintained in position by means of a horizontal wire BC attached to the axle at B and to the end C of the rod. Determine the tension in the wire and the reactions at A .



Equation Sheet For Dynamics and Vibrations PhD Qualifying Exams

Centroidal inertia properties for thin plate



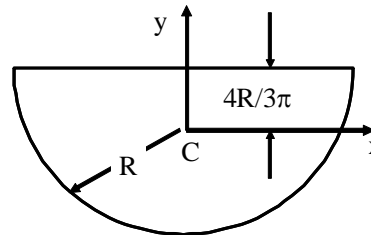
$$I_{xx} = ma^2/12$$

$$I_{yy} = mb^2/12$$

$$I_{zz} = m(a^2 + b^2)/12$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Centroidal inertia properties for semicylinder



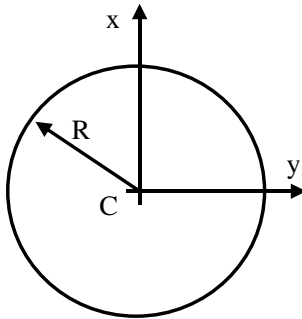
$$I_{xx} = \left(\frac{1}{4} - \frac{16}{9\pi^2} \right) mR^2$$

$$I_{yy} = \frac{mR^2}{4}$$

$$I_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2} \right) mR^2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Centroidal inertia properties for thin disk



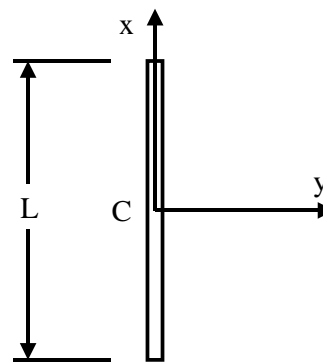
$$I_{xx} = mR^2/4$$

$$I_{yy} = mR^2/4$$

$$I_{zz} = mR^2/2$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Centroidal inertia properties for slender rod



$$I_{xx} = 0$$

$$I_{yy} = mL^2/12$$

$$I_{zz} = mL^2/12$$

$$I_{xy} = I_{xz} = I_{yz} = 0$$

Beam Flexural Vibration:

Elastic modulus E , cross-sectional area $A(x)$, cross-sectional area moment $I(x)$, density ρ , length L , transverse displacement $w(x,t)$

$$T = \frac{1}{2} \int_0^L \rho A(x) \left(\frac{\partial w}{\partial t} \right)^2 dx; \quad V = \frac{1}{2} \int_0^L E I(x) \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx$$