GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1997

Dynamics and Vibrations
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

- Please sign your name on the back of this page—

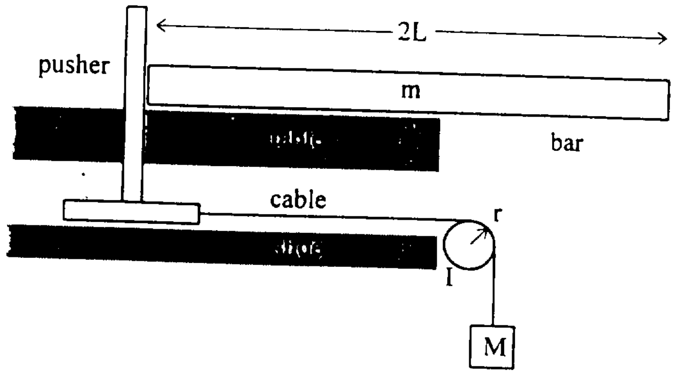
Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 1997

Instructions:

Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end.

Problem 1.

A gravity feed mechanism pushes a uniform bar along a table. The coefficient of friction is μ between the bar and the table, and between the bar and the lightweight pusher. (Neglect friction between the pusher and the slide.) A cable runs over a pulley without slipping and connects the pusher to a weight.



a) How long is required to displace the bar by distance d if the system is initially at rest?

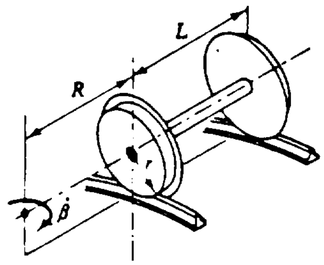
b) If the bar is pushed far enough, it will rotate off the table. What length must hang over the table to start rotation?

Problem 2.

Two identical homogeneous cylinders with small flanges are mounted on a rigid <u>massless</u> axle as shown in the figure below. The wheels are set in bearings so that each may rotate independently about the axle. The assembly is moving on a circular track on a horizontal plane as shown, and each wheel rolls without slipping. (The radius of the inner rail of the curved track is shown in the figure to be R, and the wheelset moves in such a way that the axle rotates with rate $\dot{\beta}$ about the vertical axis passing through the track center of curvature.) At a sufficiently large <u>constant</u> angular velocity $\dot{\beta}$, one of the wheels will be on the verge of lifting off the track. Recall that the mass moment of inertia of a thin disk is $mr^2/2$ about the polar axis, and $mr^2/4$ about any diameter.

(a) Which wheel tends to lift off the track?

(b) Find the angular velocity $\dot{\beta}$ for this condition in terms of g, L, r, and R.



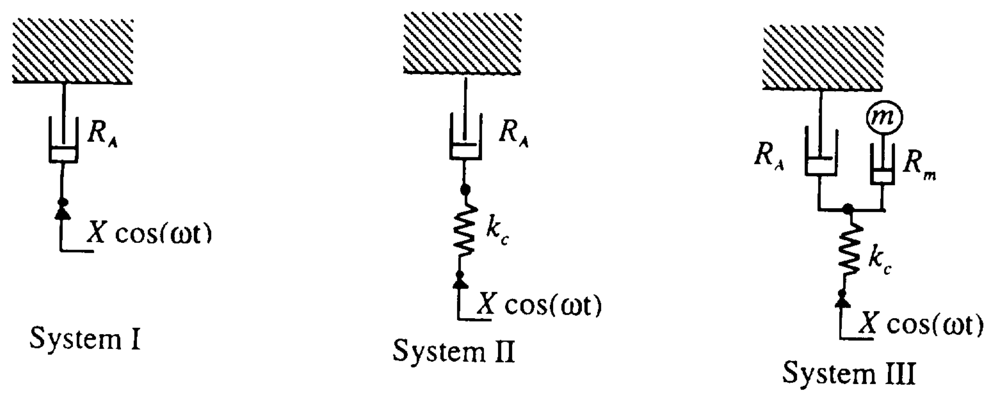
Problem 3.

The acoustic power, Π_A, radiated from a section (of area A) of the hull of a submarine vibrating with displacement $x(t)=X\cos(\omega t)$ can be modeled as being equal to the power dissipated in a damping element R_A, as shown below (System I). [$R_A=\rho cA$, where $\rho=1000 kg/m^3$, is the density of water and $c=1500 m/s$ is its sound speed.] For system I, the time averaged radiated noise is given by:

$$\Pi_A = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} R_A \dot{x}^2 dt = \frac{1}{2} R_A \omega^2 X^2$$

To reduce radiated noise, a compliant coating could be placed on the hull. If the coating material has density ρ_c and sound speed c_c and the coating has thickness *d*, at low frequencies, the effect of the coating can be modeled as a spring with spring constant $k_c = \rho_c c_c^2 A/d$ as shown below (System II). Note that there is <u>no mass</u> between R_A and k_c. To further reduce radiated noise, a mass and dashpot could be added to the surface of the coating as depicted schematically in System III.

(a) Find an expression for the radiated noise from area A when the coating is added (System II).
(b) Under what conditions will adding a coating result in a significant reduction in acoustic radiation?
(c) A mass m and damper R_m are attached to the surface A, resulting in System III below. Find an expression for the radiated acoustic power for System III.
(d) Under what conditions will System III have significantly lower radiated noise than System II.
(e) **Extra credit:** How much mass per square meter of the hull would have to be added for System III to be significantly more effective than System II at 200Hz. Is this practical?



Problem 4.

A long slender rod of length $2a$, mass m_2 and mass moment of inertia $I_c = m_2 a^2/3$ is attached to a cart of mass m_1 by means of a frictionless pin and a torsional spring k_T as shown.

(a) Find the equations of motion for the system in terms of the displacement of the cart, x(t), and the rotation of the rod from vertical, θ(t). Use any method you wish.

(b) Linearize the equations if necessary and write them in matrix form.

(c) Find the natural frequencies and natural modes of the system assuming that $k_T > m_2 g a$.

(d) What happens when $k_T < m_2 g a$?