GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1998

Dynamics and Vibrations
EXAM AREA

____

Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

Please **print** your name here.

The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 1998

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, be sure to show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave any algebra until the end. Good Luck.

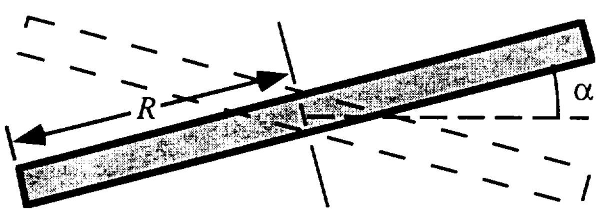
1) A silver dollar (thin disk) of radius R and mass m is tossed spinning into the air. It is observed that the dollar stays at an angle α above the horizontal. Note that the polar and transverse moments of inertia for a thin disk are $(mR^2)/2$ and $(mR^2)/4$, respectively.

a) Draw and label the precession and spin axes on the diagram below.

b) What is the angle between the angular velocity of the dollar with respect to the ground, $\bar{\omega}_{\mathcal{D}}$, and the vertical? You may find this trigonometric identity useful:

$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta \tan\phi}.$$

c) If the dollar spins at an angular speed $\dot{\psi}$, what is its rate of precession, $\dot{\phi}$?



Problem 2

A ship's turbine has a mass of 2500 kg and a radius of gyration about its axis (y_c in Figure) of 0.45 m. It is mounted on bearings as indicated and turns at 5000 rpm clockwise when viewed from the stem (rear) of the boat.

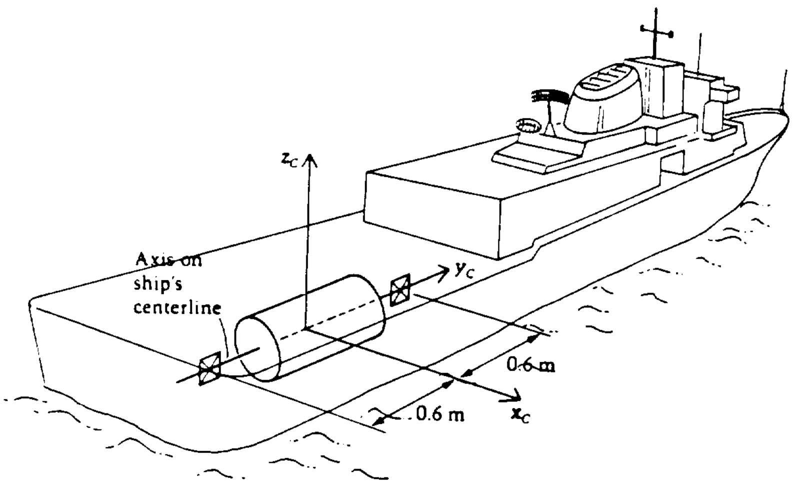
a. If the ship is in a steady turn to the right of radius 500 m and is traveling at 15 knots, what are the reactions exerted on the shaft by the bearings? (1 knot = 1.15 mph = 1.85 km/hr)

b. If the ship on a straight course in rough seas pitches sinusoidally at ± 12° amplitude with a 6-s period, what are the maximum bearing reactions then?

Note: $I_{yy} = (2500\ kg)(0.45m)^2 = 506\ kg \cdot m^2$

$I_{xx} = I_{zz} = 1000\ kg \cdot m^2$

You may assume that the turbine's center of mass coincides with that of the ship, and that the pitching motion in part (b) occurs purely about x_c.



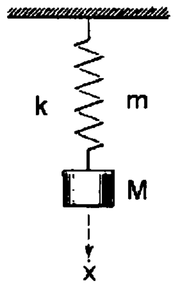
Figure

Problem 3 consists of two related parts, I and II. Answer both parts; note that part II itself consists of two questions (a) and (b).

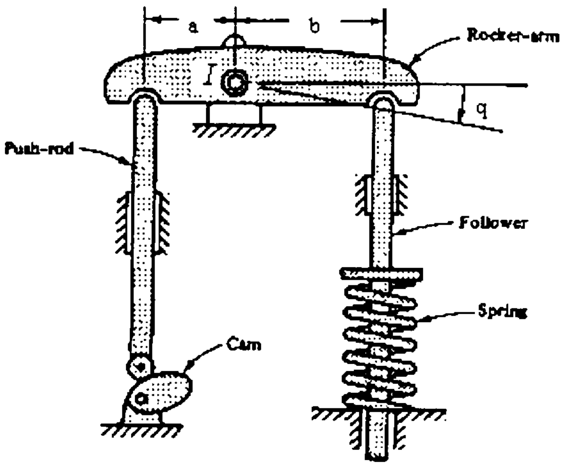
I. Show that the natural frequency for a spring-mass system can be estimated by

$$\omega = \sqrt{\frac{k}{M + m/3}}$$

What assumption(s) is (are) used for this estimation?



II. A high-speed automotive subsystem consists of a push rod, rocker arm, follower and follower spring as shown in the accompanying figure. It is driven by a cam (assumed as rigid) as shown.



The parameters of the system are

- I = mass moment of inertia of rocker arm about its axis of rotation
- m_r = mass of push rod
- m_f = mass of follower
- m_s = mass of follower spring
- k_s = spring constant of follower spring
- k_r = spring constant of push rod

a) Determine the natural frequency of the subsystem in terms of its parameters. Include the effects of the masses m_s of the follower spring and m_r of the push rod (treat as a stiff spring). Assume that because of the type of linkage connection used the upper ends of the push rod and follower both move with the contacting portions of the rocker arm, and that the axial deformation of the push rod is negligible.

b) Assume by experimentation that the (fundamental) natural frequency of the system has been determined. Do you expect the result from part a) to be higher or lower? Why?

Problem 4

A lightly-damped three degree-of-freedom system is shown below. A test engineer measured the transfer functions from the applied force $F(t)=F_0\cos(\omega t)$ to displacements x_1 and x_3. The two transfer functions, labeled Disp. A and Disp. B, are shown in the figure below. Unfortunately, the engineer failed to note which curve went with x_1 and which went with x_3.

(a) Explain which curve in the figure pertains to x_1 and which pertains to x_3. Be as persuasive as you can; no credit will be given for a guess.

(b) Give approximate values for the natural frequencies of the system.

(c) It is known that $k_2=2N/m$, $k_3=1N/m$, and all the masses are identical, $m_1=m_2=m_3=m$. Furthermore, it is known that the second modeshape is proportional to $[1\ 1\ -1]^T$. Find the numerical values of k_1 and m.

(d) As you can see, the engineer also failed to capture many of the peaks in his plot of the two transfer functions. Estimate the peak amplitude of the dashed line at the third resonant frequency. Explain your reasoning.