GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1999

Dynamics & Vibrations
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 1999

Instructions:

1. Please *solve three* of the four problems on this exam.
2. It is important that you clearly mark which three problems you wish to graded.
3. For the three problems that you select, be sure to show all work in order to receive full credit.
4. Be sure to budget your time. Concentrate on setting up the problem s[illegible] first, and leave any algebra and computations to the end.

Good luck!

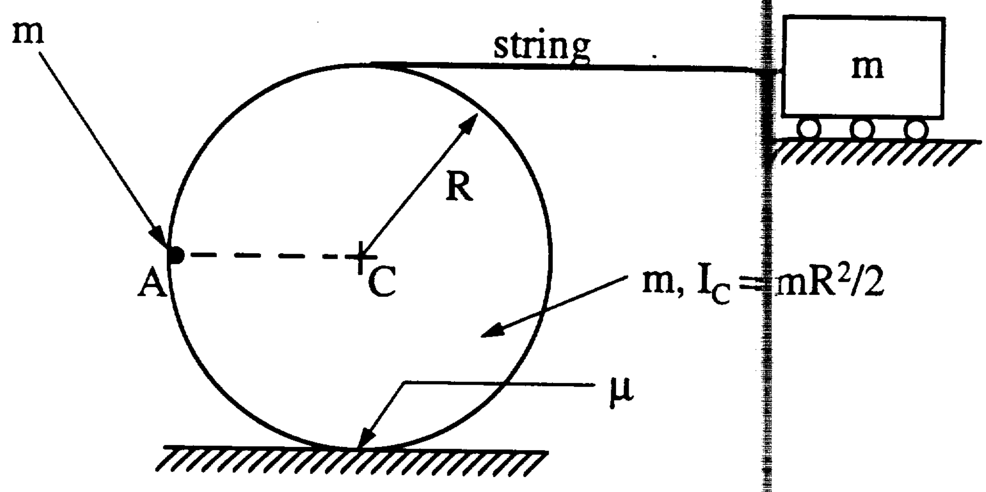
Problem 1.

A uniform disk of mass m, radius R, and mass moment of inertia $I_C = mR^2/2$, is modified by the inclusion of an *additional* point mass m as shown. The additional mass is firmly attached to the rim of the disk at point A, which is initially at the same height as the geometric center of the disk, C. A cart also having mass m is connected to the disk by means of a massless, inextensible string which is wrapped tightly around the disk's outer rim. The cart rests on rollers which have negligible mass and friction. The system is released from rest in the position shown.

(a) Find the minimum coefficient of friction, μ_{min}, between the disk and the ground such that pure rolling occurs when released.

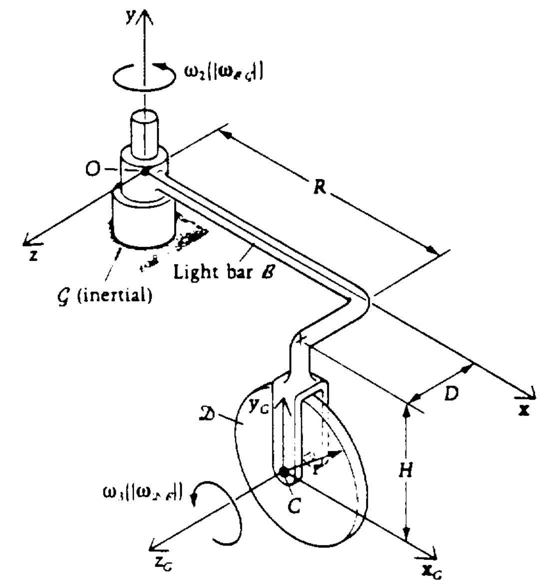
(b) Assuming that pure rolling occurs and that the string remains taught, find the velocity of the cart when the disk has rolled through 90° counter-clockwise; i.e., such that point A is in contact with the ground.

(c) If there is no friction between the disk and the ground, $\mu=0$, what conservation laws apply to the combined system? Be specific.



Problem 2

A heavy disk $\mathcal{D}$ of mass m and radius r spins at the angular rate ω_3 ($= |\omega_{\mathcal{D}/\mathcal{B}}|$) with respect to the rigid, but light, bent bar $\mathcal{B}$. (See Figure) Body $\mathcal{B}$ turns at rate ω_2 ($= |\omega_{\mathcal{B}/\mathcal{G}}|$) about a vertical axis through O, a point of both $\mathcal{B}$ and the inertial frame $\mathcal{G}$. Find the force and couple



that must be acti on $\mathcal{B}$ at O to produce a mo system for which ω_2 and ω_3 are constants. B th axes in the figure re fixed in $\mathcal{B}$, and note that are always princi l axes for $\mathcal{D}$ at C even thoug not fixed in $\mathcal{D}$.

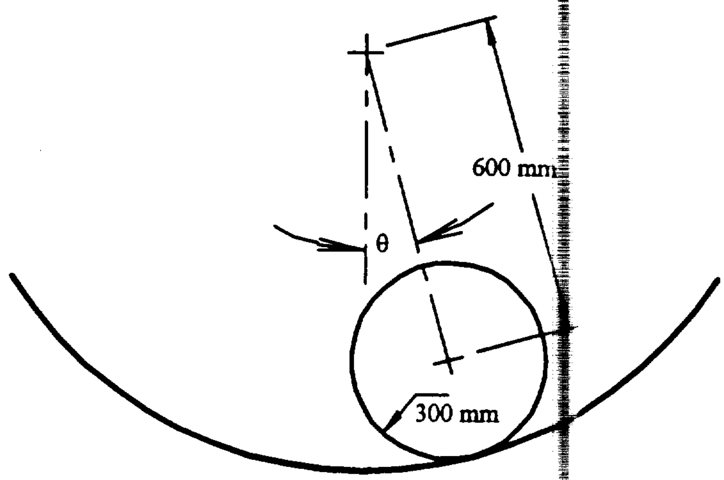
(x_G

Problem 3

The cylinder, whose mass is 2 kg, rolls without s pping in the inteior). circular valley. The moment of inertia of the cylinder bout its axis is $\frac{1}{2}mr$. system is released from rest at $\theta = 5°$.

1. Determine the elapsed time after release at which th cylinder attains the h g positon on the other side. Explain whether this ans er is exact or approx n Assume air resistance is negilgible.

2. Suppose the effect of air resistance is a force whos magnitude is propo t. to the speed of the center of the cylinder. It is o erved that after five c left and right, the highest elevation attained by t e cylinder is $\theta = 4°$, w occurs on the right side. Based on this observat n, what is the const u proportionality governing this resistive force?



Problem 4

The positions depicted in the sketch corresponds to static equilibrium wh force $F(t)$ is not present. The system lies in the horizontal plane, so gra unimportant. Dissipation also is very small. The mass of the bars are 2 kg and for the upper and lower bars, respectively. (The moment of inertia of a bar of m and length ℓ about its end is $\frac{1}{3}\sigma\ell^2$.) In an experiment the excitation $F(t)$ is har with an amplitude of 10 newtons. The following two observations are made o system's ***steady-state response*** for different frequencies:

- When the excitation frequency is very low, the amplitude of the upp bar's rotation is 0.016667 rad.
- When the excitation frequency is 3.376 Hz, the upper bar does no vibrate.

Determine the values of the spring constants k_1 and k_2 that will lead to these tions. Then determine the frequencies at which each bar's rotational amplitu be largest.