Dynamics and Vibrations Ph.D. Qualifying Exam
**Spring 2006**

Instructions:
Please work 3 of the 4 problems on this exam. <u>It is important that you clearly mark which three problems you wish to have graded</u>. For the three problems that you select, show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

The figure models a landing gear on an airplane. The wheel spin ω_s and the lifting velocity ω_x are assumed constant. State any important assumptions to complete the following:

  a) Determine the net force on the wheel.
  b) Determine the net moment on the wheel.
  c) Determine the moment M_0 applied by the lifting motor.
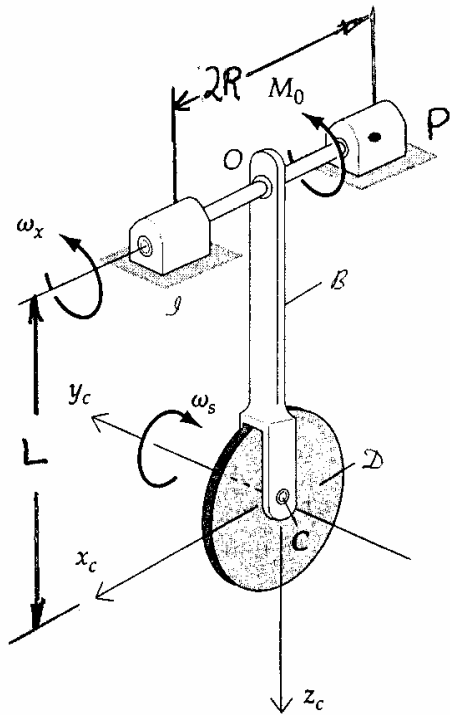
Problem 2.

Two inverted simple pendulums are connected together. While the lower pendulum can swing freely in the 2D plane, the upper mass is restricted to vertical motion in a viscous guide, where the spring is unstretched in the position shown below. Note that because of the guided restriction, the system is sufficiently defined by single degree of freedom. The stiffness k is such that static deflection under the action of gravity is occurring at $\theta=\pi/4$. Now, after the system has deflected statically to $\theta=\pi/4$, a very small vertical force $F(t)=F_o sin(\omega t)$ is acting upon the upper mass, where the F_o is the magnitude of the force and ω is the forced frequency. Assume small oscillations about the static equilibrium. Do the following:

(a) Determine the value of k such that static deflection under the action of gravity is occurring at $\theta=\pi/4$.

(b) Write the equation of motion in terms of θ (relative to the static equilibrium of the system), and linearize the equation as appropriate,

(c) Determine the steady state response of the system.
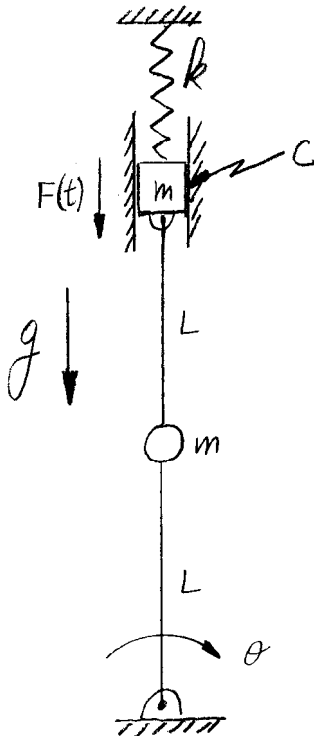
Problem 3.

A non-uniform rigid rod of mass, m, length, L, and mass moment of inertia, $I_G = m\alpha^2$, is pinned at its midpoint, G, as shown. The radius of gyration for the non-uniform rod is $\alpha = \sqrt{2}L/2$. A spring-mass system is attached at the left end of the rod and the right end is excited by a prescribed displacement, $y(t)$, applied to the base of another spring of stiffness, k.

(a) Find the equations of motion in terms of the generalized coordinates $x(t)$ and $\theta(t)$. Use small angle approximations, and assume that both coordinates are measured from static equilibrium values (with $y = 0$).
(b) Find the natural frequencies of the system in terms of k and m.
(c) When the right-hand spring is excited at a frequency that is 0.99 times the first natural frequency, find the force in the left-hand spring if the amplitude of the mass is $x = L/10$.
(d) If $y(t) = Y\sin(\omega t)$, **sketch** the amplitude ratio of θ to y as a function of ω. Your sketch should pay special attention to the low-frequency limit, the high-frequency limit, and any points of large or zero response.
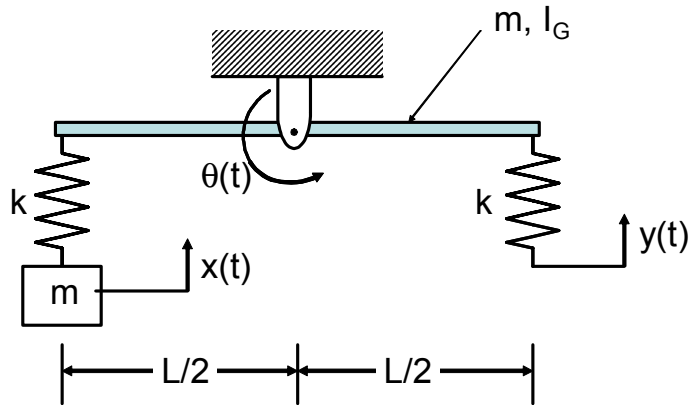
Problem 4.

A Block B of mass M can translate freely on a frictionless horizontal and stationary surface which contacts B's lower face. At B's center of mass G, a mathematical pendulum P of mass m and length L is attached. The pendulum is given an initial angular displacement ψ_0 from rest and is then released.

(a) Express the generic displacement of G x (measured from G's initial position, say, positive to the right) as a function of P's (generic) angle ψ; i.e., find:

$$x = Function(\psi : \psi_0, M, m, L)$$

(b) What is the maximum speed attained by Block B?