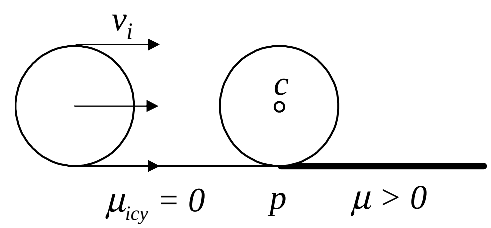
Instructions: Complete 3 out of the 4 following problems. Only the first 3 problems handed in will be graded.

1. A free (unattached) wheel of mass $m$, radius $r$, and inertia $I_c = \frac{1}{2}mr^2$ is initially translating with initial velocity $v_i$ on an icy road with $\mu_{\text{icy}} = 0$ and contact point $p$. At time $t = 0$, the wheel comes into contact with a rough road with $\mu > 0$. Over this portion of the road, the wheel starts to also rotate which later, at time $t = t_r$, turns into pure rolling. Determine time $t_r$ in terms of the given information and the gravity constant $g$.



2. Consider a rigid, homogeneous, and thin square plate, of mass $m$ and edge length $a$, one corner of which is freely pivoted to a fixed point $O$ of a smooth horizontal plane. Initially, the plate rests vertically and then falls under gravity, with its lower edge $OA$ sliding on the plane. *Using the well known conservation theorems of dynamics*, find the equations satisfied by its Eulerian–type angles $\phi$ and $\theta$, i. e. obtain a system of *two* (coupled) *first order* differential equations for $\phi(t)$ and $\theta(t)$ (see Figure). Last, discuss methods of simplification (uncoupling) & integration of these equations; no need for actual further integration.

**Additional Data & Hints:**

i) Choose as *fixed* (inertial) axes $O\text{–}xyz$, such that $Ox$ is the initial position, on the plane, of the plate edge $OA$, and $Oy$ also lies on that plane, while $+Oz$ is vertically upwards.

ii) Angles: $\phi$: angle (of precession) formed by plate edge $OA$ with $+Ox$,

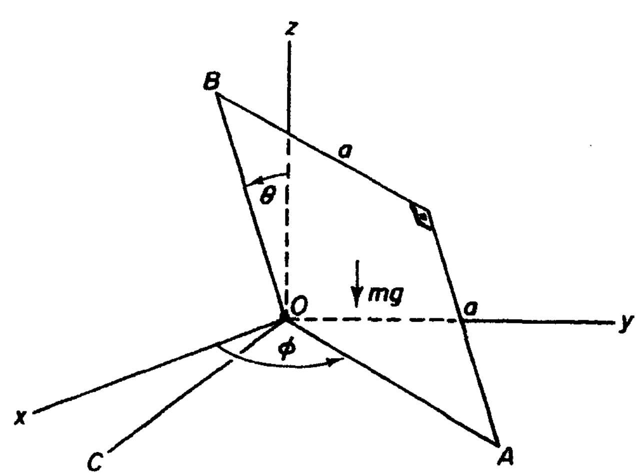
$\theta$: angle (of nutation, or) inclination of plate to $Oz$

iii) Choose as *moving* (non–inertial) axes the plate–fixed axes $O\text{–}ABC$ (see Figure).

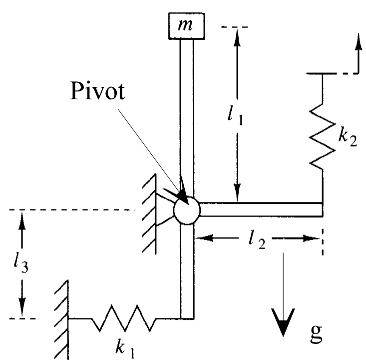
iv) Relative to the latter, the plate's inertia tensor (at $O$) has the following components:

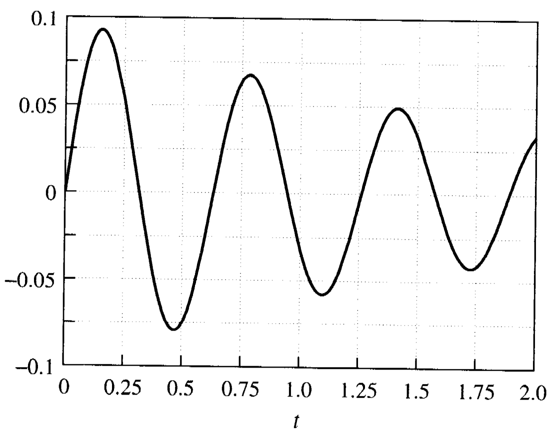
$$I_{AA} = I_{BB} = (1/3)\, m\, a^2, \qquad 1. \qquad I_{CC} = (2/3)\, m\, a^2;$$

$$I_{AB} = -(/1/4)\, m\, a^2, \qquad I_{AC} = 0, \qquad I_{BC} = 0.$$



A time dependent displacement $y(t)=y_o cos(\omega t)$, is applied at the free end of spring 2 (see figure below). The two rods can rotate sideway in the plane of the paper about the pivot. There is some viscous damping at the pivot caused by bearing friction. An experimental natural response of the system is shown below. Based on the system geometry ($l_1=1\ m$, $l_2=l_3=0.5m$) and stiffness parameters ($k_1=k_2=220\ N/m$) determine to the best of your ability the system mass, the damping coefficient in N-m-s/rad, and the maximum rotation angle of the forced response, letting $y_o=10\ mm$ (also estimate at which ω the system response is maximum). Assume the rods to be massless, and that gravity is acting downwards.



4. A block of mass *m* and a cylinder of mass *m* and radius *R* are connected by 3 identical springs of stiffness *k* as shown. The mass moment of inertia of a uniform cylinder is $I = mR^2/2$.

(a) Assuming that there is sufficient friction between the cylinder and the ground to prevent slipping, find the natural frequencies and natural modes of the system.

(b) Consider a harmonic force of frequency ω and magnitude $F_1 = mg/2$ applied to the first mass. Find the frequency range over which pure rolling can be maintained if the coefficient of friction between the cylinder and the ground is $\mu = 0.2$. Concentrate on developing the correct expressions, and solve them only if time permits. To simplify the algebra, you may assume that $m=1\ kg$ and $k=1N/m$.