Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2009

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

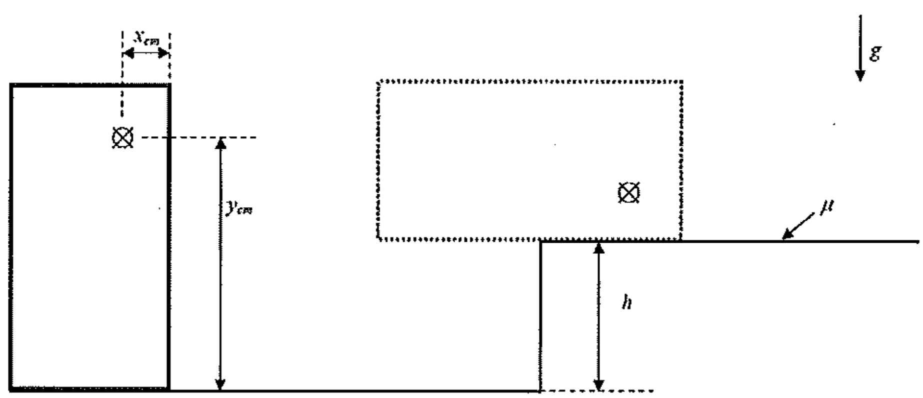
Problem 1.

The box shown is made of a heterogeneous material such that the center of mass is located in the top right corner as shown. **Note that the products of inertia are zero for this box.** The mass is given as m and the moment of inertia about the mass center and about the z-axis (out of the paper) is given by I_{zz}.

Initially the box slides on the surface heading to the right. Some time later it strikes a step in the surface of height h with a speed v. Assuming the impact between the box and the step is perfectly plastic, and that friction is large-enough such that the box does not slip or lose contact with the step's corner during the ensuing rotational motion,

(i) **find the speed v required in order for the box to rotate exactly through 90 degrees such that it hits the top of the step with zero rotational speed.** The final configuration is shown as the box laying on the surface (dotted-line box). Express your answer for v in terms of m, I_{zz}, g, x_{cm}, y_{cm}, and h.

(ii) comment on the feasibility of the solution – i.e., what is required for the box to land horizontally on the step with zero rotational speed.

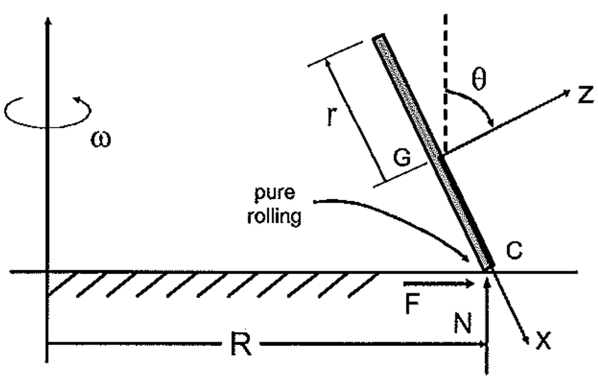


Problem 2.

A homogeneous hoop of mass m and radius r rolls without slipping on a fixed and rough horizontal surface. (Note: the figure depicts an edge-on view of the hoop.) The mass moments of inertia of the hoop at its mass center G (which is also its geometric center) are $I_{xx} = I_{yy} = 0.5mr^2$, $I_{zz} = mr^2$, relative to the xyz-axes shown. The mass center G of the hoop moves in a circle about a stationary vertical axis such that it completes one revolution every T seconds, and such that the contact point traces a circle of radius R; note that this implies θ = const.

(a) Find the relationship between θ, T, m, r, R, and gravitational accel. g required for this motion to be possible.

(b) Find the minimum value of the friction coefficient such that slip will not occur at point C.



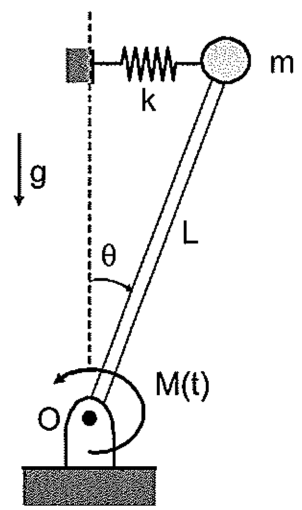
Problem 3.

Consider the inverted pendulum shown in the figure below. The rod is massless, and is restrained by a fixed, frictionless pivot at point O. Gravity acts downward.

(a) Determine the small-angle equation of motion, and discuss its stability when $M(t) = 0$.

(b) For the conditions in part (a) for which the solution is unbounded, determine the conditions on an imposed, moment $M(t) = a\theta + b\dot{\theta}$ that would yield a bounded response.

(c) For the case where $M(t) = A\sin(\omega t)$ and $k = 2mg/L$, find the steady-state amplitude of response for θ. Discuss how you would find the range of ω such that the small angle approx used in (a) is justified.



Problem 4.

A massless, rigid rod of length L connects 2 point masses, m_1, and m_2, The rod sits on two springs as shown in Fig. 4.1

(a) Using the coordinates y_1 and y_2, find the natural frequencies and natural modes (eigenvectors) of the system. Sketch the modeshapes of the system by drawing the rod at a particular 'snapshot" as it vibrates purely in mode 1 or mode 2.

(b) If the coordinates z and θ were used to model the system, what would the new natural frequencies and natural modes (eigenvectors) be? Express the modes in terms of 2x1 vectors $[\ z\quad \theta\ ]^T$.

(c) If $m_1 = m_2$ and $k_1 = k_2$, what are the natural frequencies of the system? What interesting property do the modes of the system exhibit in this case?

(d) Consider the case shown in Fig 4.2, where a spring-mass system k_3-m_3 is attached to the rod at a location a. (Assume again that $m_1 = m_2$ and $k_1 = k_2$) Determine the equations of motion of the system in terms of the coordinates y_1, y_2, and y_3.

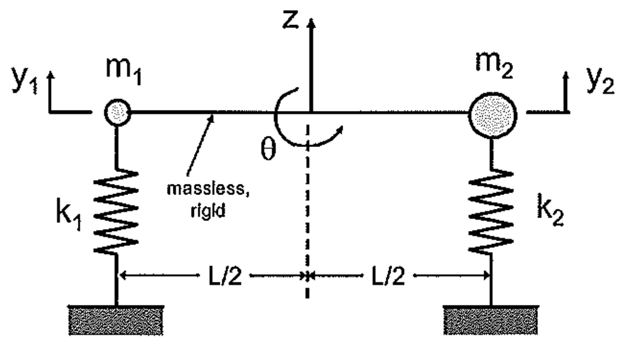


Fig. 4.1

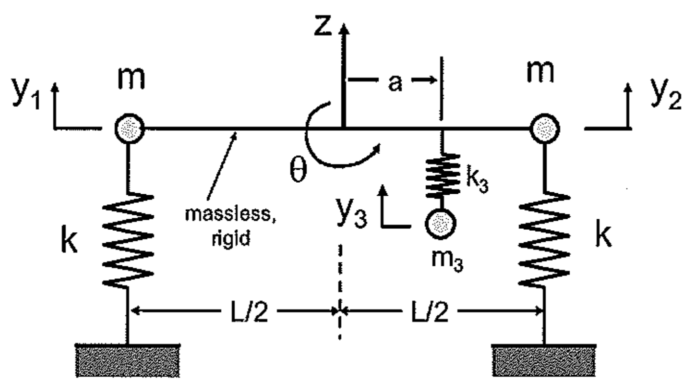


Fig. 4.2