Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2011

Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

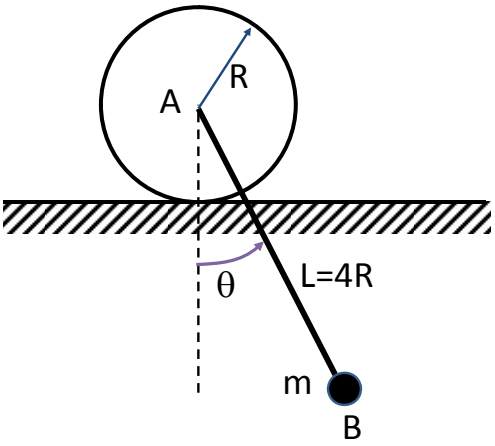
Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

A disk of mass *m* and radius *R* is welded to a thin, massless rod AB of length L=4R. A point mass *m* is located at end B of the rod.

(a) If the system is released from rest such that arm AB makes an angle θ with the vertical, find the instantaneous acceleration of the mass, $\vec{a}_B$ upon release of the system. Assume that the disk rolls without slipping against the ground.

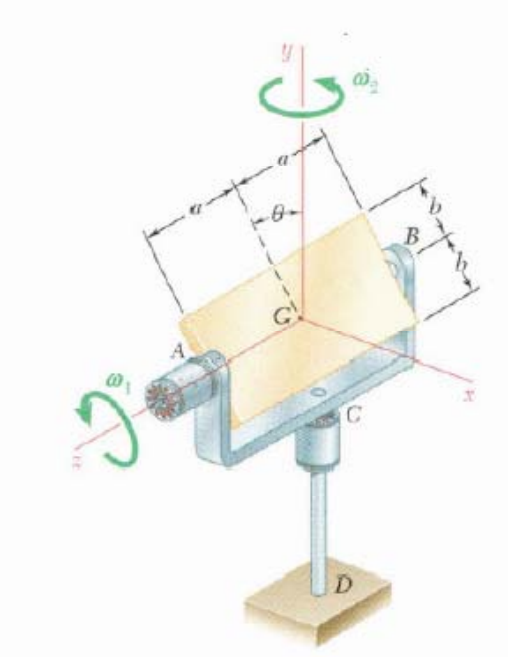
(b) Find the velocity of the mass $\vec{v}_B$ at the point when θ = 0.



Problem 2.

A 48 kg advertising panel of length $2a$ = 2.4 m and width $2b$ = 1.6 m is kept rotating at a constant angular rate ω_1 about its horizontal axis by a small electric motor attached at *A* to frame *ACB*. This frame itself is kept rotating at a constant angular rate ω_2 about a vertical axis by a second motor attached at *C* to the column *CD*. Knowing that the panel and the frame complete a full revolution in 6 sec and 12 sec, respectively, express, as a function of the angle θ, the dynamic reaction exerted on column *CD* by its support at *D*.

Note: the moments of inertia of a thin rectangular plate of width *w* and height *h* are given as $m(w^2 + h^2)/12$, $mw^2/12$, $mh^2/12$



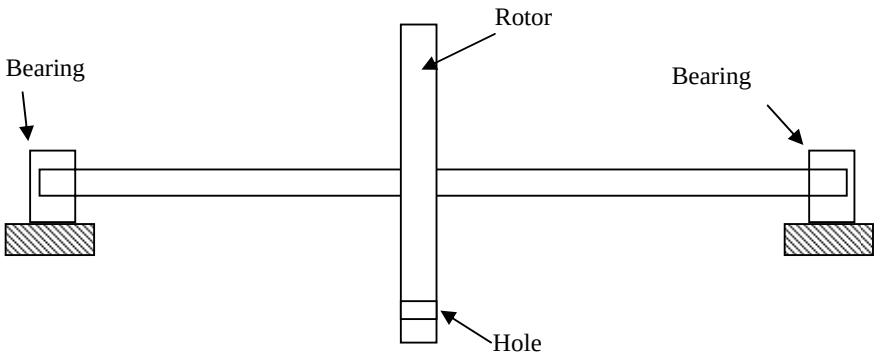
Problem 3.

Consider the system below. It is comprised of a rotor disk mounted midspan on a uniform steel shaft. The mass of the disk is 15 kg and its diameter is 0.3 m. The disk has a circular hole of diameter 0.03 m whose center is at a distance of 0.12 m from the center of the rotor. The bending stiffness of the shaft is EI=1600 Nm^2. The overall length of the shaft is 0.4 m Assume the shaft bearings are rigid. Neglect the mass of the shaft. The equivalent stiffness of the k=192EI/L^3.

(a) Write the equation for the transverse displacement of the shaft at the position of the rotor for arbitrary rotation speed ω.

(b) Determine the amplitude of vibration if the rotor rotates with an angular velocity of 6000 RPM.

(c) Write the particular solution (forced response) for the equation of motion for the case where the shaft is rotating at the natural frequency.



Problem 4.

A 2 DOF system rests motionless on a frictionless surface as shown. At time $t = 0$, a bullet of mass m_b and velocity v_b impacts the left mass resulting in a perfectly plastic collision. The numerical parameters are $m_1 = 9.9$kg, $m_2 = 10$kg, $k_1 = 20$N/m, $k_2 = 30$N/m, $m_b = 0.1$kg, and $v_b = 100$m/s.

(a) Find the natural frequencies of the system after the bullet is embedded in the left mass. Show your work.

(b) Give a closed-form expression for the motion of the left mass for $t > 0$. Neglect damping.

(c) If a small amount of modal or proportional damping is added to the system, make a sketch of the response $x_1(t)$ vs t, pointing out any important attributes.