COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Spring 2013

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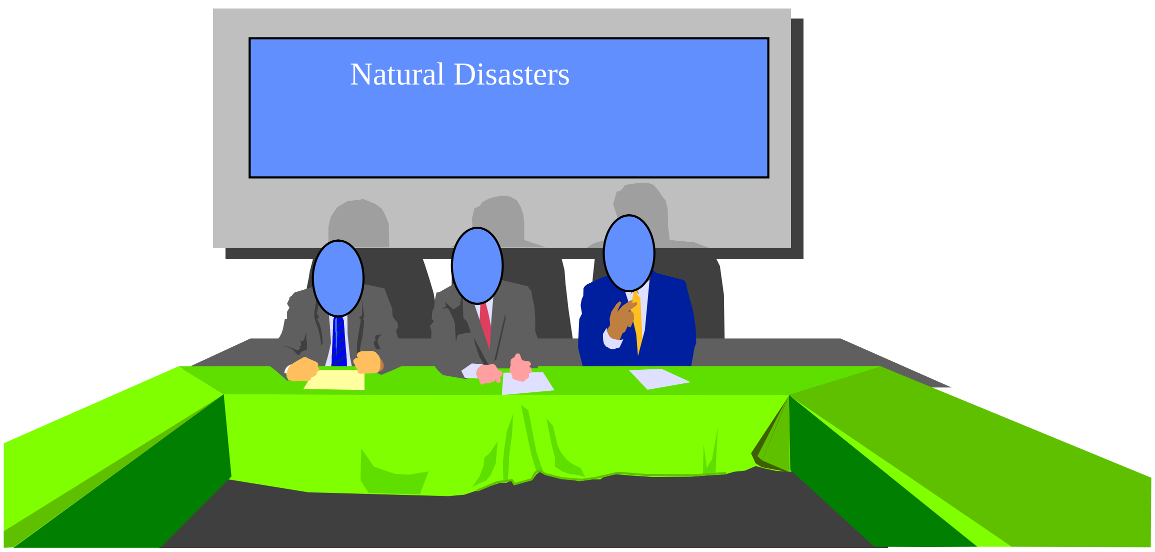


- All questions in this exam have a common theme: ***Design against Hurricanes and Natural Disasters***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

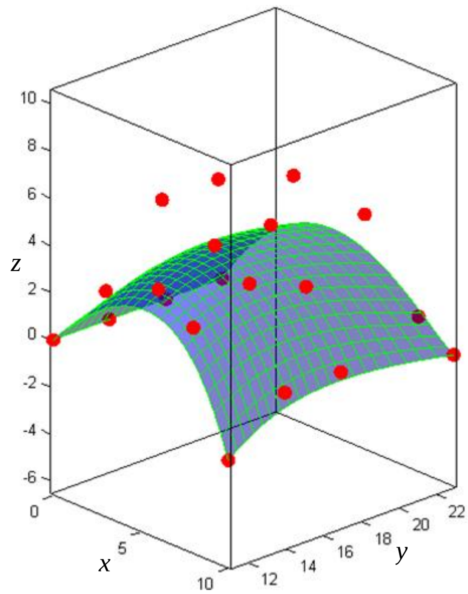
Question 1 - Geometric Modeling

Suppose that you are a design engineer in a heavy equipment manufacturing company that builds snow plows. Your current task is to design the blade for a truck.

You divided the blade into several segments and wanted to use the 3^{th} order by 4^{th} order Bézier surface patch to model.

a) Derive the equation of the 3^{th} order by 4^{th} order Bézier surface patch in a matrix form.

b) You came up with the design of one patch shown below with the control points of



Patch 1:

(0 20 0); (3 21 5); (6 22 4); (8 23 0); (10 23 -1);
(0 17 0); (3 17 6); **(6 17 5)**; (8 17 3); (10 17 0);
(0 14 0); (3 14 6); (6 14 5); (8 14 4); (10 14 0);
(0 11 0); **(3 11 3)**; (6 11 4); (8 11 3); (10 11 -2);

To start the design of the second patch, we have decided some of the control points as follows.

Patch 2:

(-10 20 0); (-7 21 -5); (-3 22 -4); P_3; (0 20 0);
(-10 17 -2); (-7 17 -6); (-3 17 -5); P_4; P_1;
(-10 14 -1); (-7 14 -6); (-3 14 -5); P_5; P_2;
(-10 11 0); (-7 11 -3); (-3 11 -4); P_6; (0 11 0);

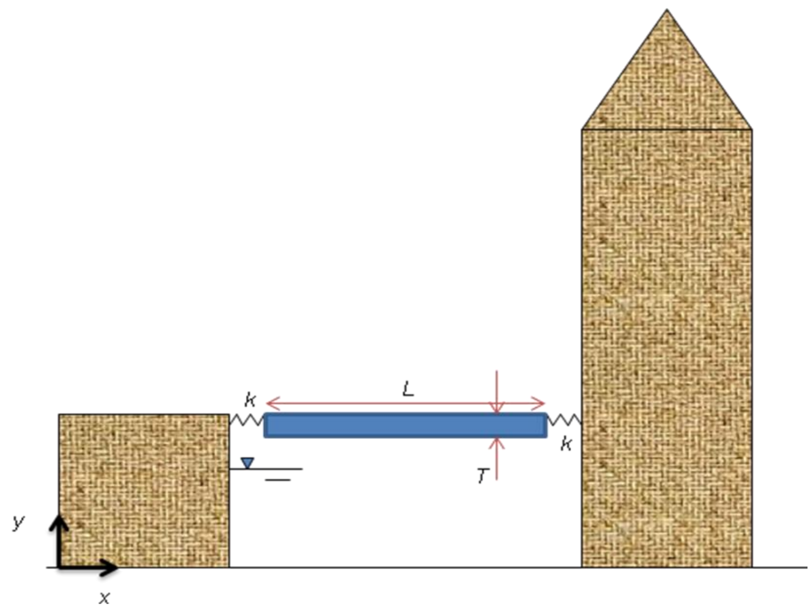
To ensure the G^1 continuity between the two patches, what are the coordinate values of $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$, $\mathbf{P}_4$, $\mathbf{P}_5$, and $\mathbf{P}_6$? Explain how you decide the values in detail and show calculations.

c) If you change one of the control point with coordinates (**6, 17, 5**) in Patch 1 to a new position (**6, 17, 10**), explain (1) how this change will affect the shape of Patch 1 and why; (2) how this change will affect the shape of Patch 2 and why; and (3) how this change will affect the continuity between the two patches and why.

d) If you change a different control point with coordinates (**3, 11, 3**) in Patch 1 to the position (**3, 11, 6**), explain (1) how this change will affect the shape of Patch 1 and why; (2) how this change will affect the shape of Patch 2 and why; and (3) how this change will affect the continuity between the two patches and why.

Question 2 – Finite-Element Analysis

The schematic shows a walkway supported by two spring-like structures over a pool of water. The spring-like structures have a stiffness k. The walkway is of length L and has a thickness T and a width W. The walkway is made of a material with a modulus of elasticity E. Under normal weather conditions, the water level is way below the walkway. However, under heavy rain, the water will rise. Assume that the water has risen to the level so as to touch the bottom of the walkway and but not submerge the walkway. Also, due to the current wind conditions, the water moves sideways along x axis and applies a uniformly distributed horizontal force f along x axis at the bottom of the walkway. In this current analysis, the focus is on the horizontal deformation of the walkway and the springs, not the vertical deflection.



Using an appropriate finite-element formulation, determine the horizontal deformation of the spring-like structures. State all assumptions clearly.

- Show all your calculations.
- Show the boundary conditions and loading conditions.
- Write down element stiffness matrix and assembly stiffness matrix.
- Determine the horizontal deformation of the springs.

<u>Element A - Stiffness Matrix</u>

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; l and m are direction cosines of the element with respect

<u>Element B - Stiffness Matrix</u>

$$[K] = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where E, I, and h are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Methods

A meteor streaked through the skies above Russia's Urals region on Feb. 15, 2013 before exploding with a flash and boom that shattered glass in buildings and left about 1,000 people hurt. The term "meteor" refers to the streak of light caused by a piece of space debris burning up in the atmosphere. The pieces of debris are called meteoroids, and remnants of the debris that reach Earth's surface (or another planet's) are called meteorites. Assuming a meteoroid falls perfectly vertically towards Earth, its behavior during can be described by the equation:

$$\frac{d^2y}{dt^2} = -\frac{1}{2}g + \mu\frac{v^2}{m}$$

where y is the distance from the ground (vertical), g is the acceleration of gravity, v is the meteoroid's speed, m is the mass of the meteoroid, and μ is the friction coefficient.
Assume that g=10 m/s^2, m=1.0 kg, and μ =0.0001 kg/m. The initial conditions are y_0=5,000 m, and v_0=10 m/s.

(a) An initial value problem is given by $y' = f(x, y)$, $y(x_0) = y_0$. Let $y = \phi(x)$ be the solution such that $\phi(x_{n+1}) - \phi(x_n) = \int_{x_n}^{x_{n+1}} f(t, \phi(x))dt$. Identify a recursive formula to approximate y by applying the trapezoidal rule. *Note: the derived solution is called the improved Euler method* (*Heun's method*).

(b) Write down the two first-order equations (state space representation) for the meteors behavior.

(c) Determine approximate values for the solution, y at time t=0.4. Use the improved Euler method with step size h=0.2.

(d) Assume the numerical error of your solution from (c) is approximately 2 m. If instead you were to iterate again until 0.4 seconds but with a time step of 0.02 rather than 0.2, what would be a good estimate of the numerical error? Justify your answer.

(e) To predict the impact time and impact speed of the meteoroid, you would have to simulate for about 50 seconds. Assume that in this case you would use a time step of 2 seconds. Comment on the accuracy of your simulation in this scenario.

Note: The Euler formula is $y_{n+1} = y_n + hf_n$

The trapezoidal rule is $\int_a^b f(x)dx = (b-a) \times \frac{f(a)+f(b)}{2}$