COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Fall 2005

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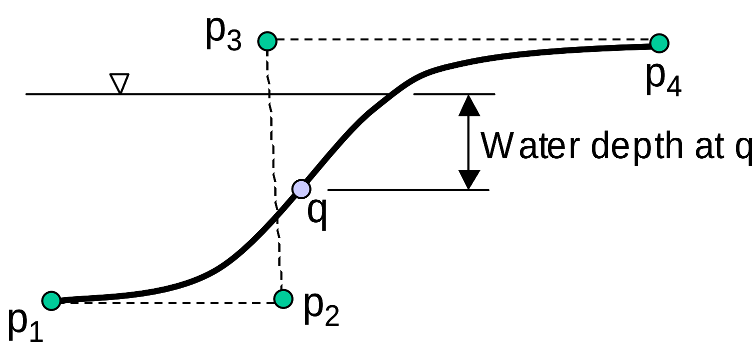
- All questions in this exam have a common theme: ***Design against Hurricanes and Natural Disasters***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

A system of levees is supposed to protect New Orleans from flooding. Assume that a typical levee has a profile that is described by a cubic Bezier curve, as shown below, where the curve is defined by control vertices p_1 to p_4. Equations for Bezier curves are also shown below.



$$b(u) = \sum_{i=0}^{n} B_{i,n}(u)\vec{P}$$

$$B_{i,n}(u) = \binom{n}{i} u^{i}(1-u)^{n-i}$$

Assume that the control vertices have the following values (units are feet). Answer the questions.

$\mathbf{p_1}$ = (0, 0), $\mathbf{p_2}$ = (20, 0), $\mathbf{p_3}$ = (15, 25), $\mathbf{p_4}$ = (50, 25).

a. Derive the equation of the cubic Bezier curve that models the levee.

b. Compute the coordinates of the point on the Bezier curve at u = 0.4.

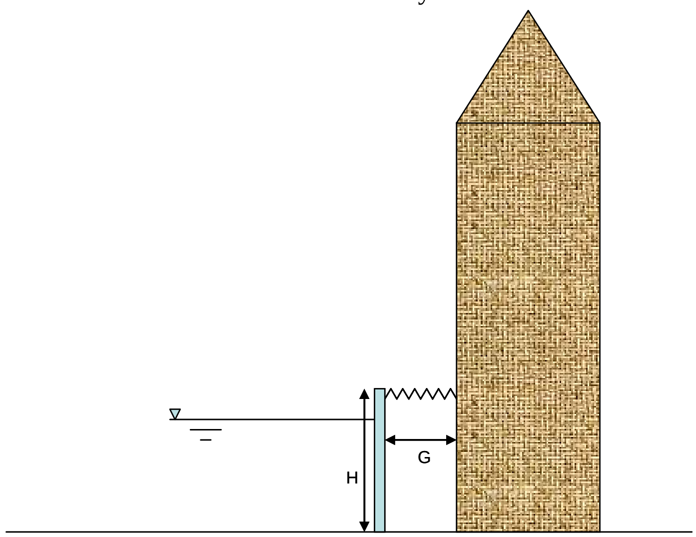
c. If the flood waters are 20 feet high, relative to the base of the levee, what is the force on the point q from the water? Assume water weighs 62 lb/ft^3. State any assumptions that you have to make.

d. If the flood waters are 20 feet high, compute the total force (weight of water) on the submerged portion of the levee. Describe how you will formulate this problem and your approach to solving it. Then, solve it if you have time.

e. Explain how you would approach computing the force on the levee from a 5 foot high wave that is traveling horizontally.

Question 2 – Finite Element Analysis

To prevent damage to a tall building from water surge, a vertical structure with a height H is erected at a distance G from the building as shown in the schematic below. The vertical structure has a width *W* and a thickness *T*. The material used in the vertical structure has a modulus of elasticity *E*. To ensure that the vertical structure will not fail when the water level reaches the height of the vertical structure, additional reinforcement in the form of springs are also installed near the top of the vertical structure as shown in the figure. The reinforcement springs have a total stiffness of *K*.



Using an appropriate finite-element formulation, determine the horizontal deflection of the vertical structure near the top. For the sake of simplicity, you may assume that the total force due to water pressure is applied horizontally at 1/3rd height of the water level.

- State all assumptions clearly.
- Show all your calculations.
- Show the boundary conditions and loading conditions.
- Write down element stiffness matrix and assembly stiffness matrix.
- Determine the horizontal deflection at the top of the vertical structure when the water level is *H*. The design team is not interested in the stress/strain distribution in the vertical structure

Element A - Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

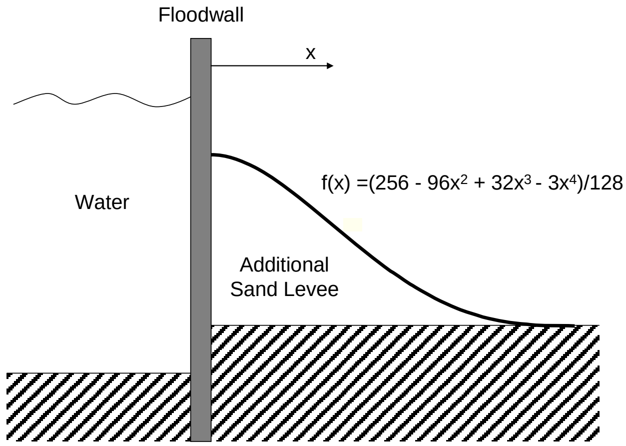
where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines of the element with respect to *X* and *Y* axes.

Element B - Stiffness Matrix

$$[K] = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E*, *I*, and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Methods



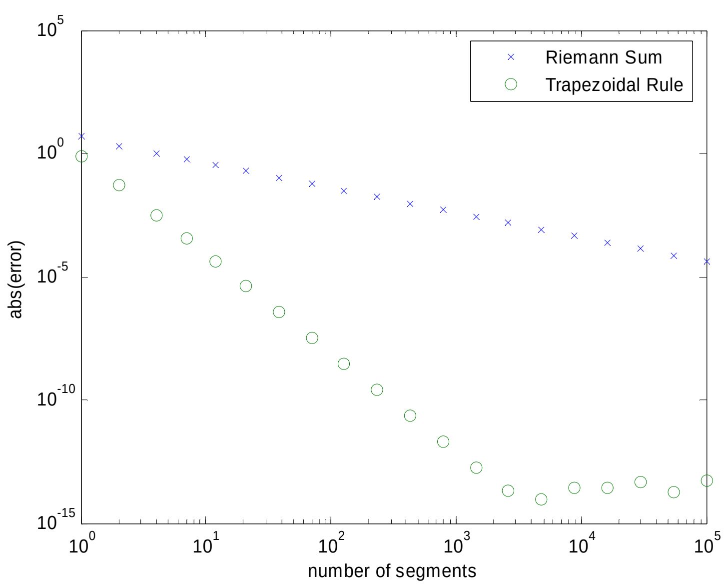
The day after hurricane Katrina passed New Orleans, several floodwalls in the city collapsed under the pressure of the rising water. The concrete floodwalls had been built higher than specified by the Army Corps of Engineers causing their foundations to give way. The Army Corps of Engineers is now considering different options for strengthening the remaining floodwalls such that similar catastrophes can be avoided in the future.

One option under consideration is to add an additional sand levee that would further support the concrete wall. Engineers have determined that the sand levee should follow a profile specified by the 4th order polynomial in the figure above. The value *f(x)* specifies the height of the additional sand (in meters) that needs to be deposited on top of the current ground level. According to this equation, the additional levee will extend 4 meters to the right of the floodwall (the distance *x* is measured in meters starting from the floodwall). To evaluate the viability of this potential solution, you are asked to compute the volume of sand that is required per running meter of floodwall.

Questions:

a) Using Simpson's 1/3 rule, compute the volume of sand per running meter of floodwall. (use a segment size of 1).

b) We computed the same sand volume using both a Riemann Sum and the Trapezoidal Rule. In the figure below, the errors of these computations are plotted as a function of the number of integration segments. Interpret the figure.



c) If you were to choose a numerical integration algorithm, which performance characteristics of the algorithm would you consider in selection? Why are there multiple algorithms? Is there not a single best algorithm? Explain.