COMPUTER-AIDED ENGINEERING
Ph.D. QUALIFIER EXAM – Fall 2007

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- All questions in this exam have a common theme: ***The America's Cup Sailing Race***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1:
Geometric Modeling

In this problem, we will investigate the shape of an America's Cup yacht cross-section. Assume that most of the hull's cross-section can be modeled using two cubic Bezier curves, as shown in the plot below. The coordinates of the control vertices (CVs) are:

Curve A: X = 200, 240, 160, 60

Y = 200, 60, 20, 10

Curve B: X = 60, ??, 12, 10

Y = 10, ??, -40, -80

a) Compute the coordinates of the second CV on curve B, assuming C1 continuity.

b) Compute the point on curve B at u = 0.5, given your computed CV.

c) Sketch the two Bezier curves for the boat's cross-section, including the CVs.

d) Do curves A and B have C1 continuity? Explain why or why not.

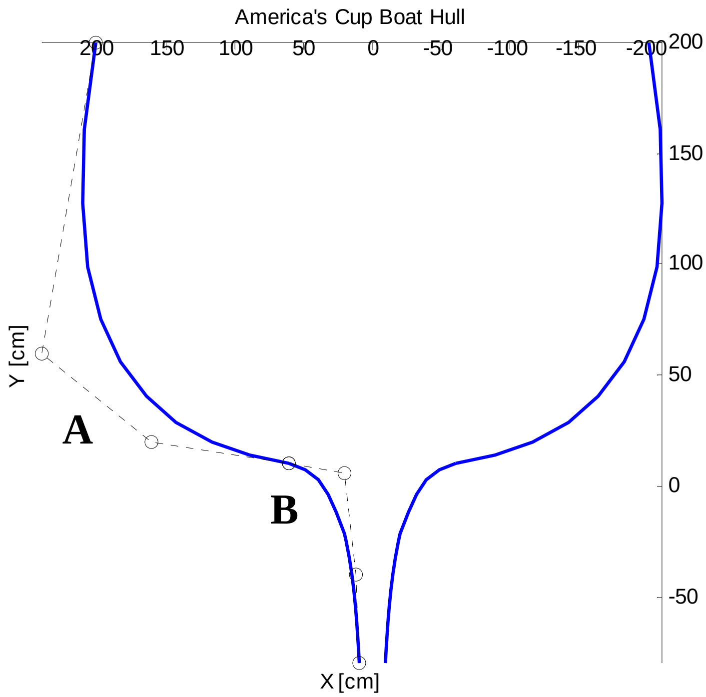
e) Boat cross-sections should be mirror symmetric. Develop the transformation matrix for mirroring the Bezier curves to generate the right-hand side curves.

f) Apply the mirror transformation matrix to the CVs for Curve A to compute the CVs for the right-hand side curve.

Bezier curve equations:

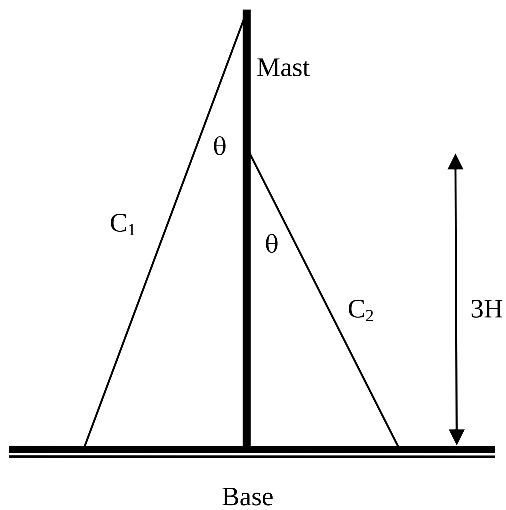
$$b(u) = \sum_{i=0}^{n} \underset{\sim}{p}_i B_{i,n}(u)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$



Question 2:
Finite Element Analysis

In the early stages of the design, a team is performing an analysis to understand the effect of cable tensile force on the mast of a boat. Accordingly, the team is considering a simplified model of the mast. In the model, the mast is assumed to be vertical with a height of *4H* and circular with a cross-section radius of *r*. The mast is made of a material with a modulus of elasticity *E*. The design team assumes that the mast is rigidly held at the base as shown in the figure. Two cables – C1 and C2 - are assumed to be attached to the mast at the tip and at a height of *3H* from the base, respectively, as shown. The other end of the cables is attached to the base. The cables make an angle θ with the mast, and each cable exerts a tensile force of *F*. You are asked to determine the mast tip displacement under the tensile force from the two cables.



1. Use finite-element formulation to solve for the displacement at the mast tip.
2. State all of your assumptions clearly.
3. Show the boundary conditions and loading conditions.
4. Starting with the element matrices, write down the assembly stiffness matrix.
5. Show all steps to determine the mast tip displacement. You need not solve.

Element A - Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

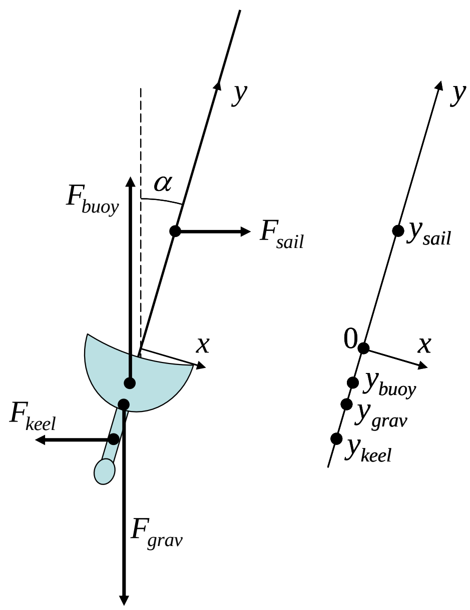
where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines of the element with respect to *X* and *Y* axes.

Element B - Stiffness Matrix

$$[K] = \frac{2EI}{L^3}\begin{bmatrix} 6 & -3L & -6 & -3L \\ -3L & 2L^2 & 3L & L^2 \\ -6 & 3L & 6 & 3L \\ -3L & L^2 & 3L & 2L^2 \end{bmatrix}$$

where *E*, *I*, and *L* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Problem 3: Numerical Methods



When designing a sailboat, it is important to make sure that the boat does not capsize too easily. The rolling (side-to-side) motion of a sailboat is determined by the four forces illustrated in the figure on the right. With the wind blowing from the left, the boat experiences a force on its sail given by F_{sail}. Through the interaction with the water, the keel at the bottom of the boat provides an equal but opposite force, F_{keel}, which stops the boat from accelerating sideways. Similarly, in the vertical direction, the gravity force, F_{grav}, is opposed by a buoyancy force, F_{buoy}, of equal size but opposite direction. In this exam question, the focus is on how these forces affect the side-to-side rotation of the boat. In steady state conditions with a wind of 10m/s, the four forces result in a zero torque only when the boat is listing ("listing" = "leaning to the side") at an angle α. Your task is to compute this angle α, given the following equations for the forces:

$F_{sail} = F_{keel} = 3900\cos(\alpha)$ applied at a location on the y-axis equal to: $y_{sail} = 5$ and $y_{keel} = -3$

$F_{grav} = F_{buoy} = 30,000$ applied at a location on the y-axis equal to: $y_{grav} = -2$ and $y_{buoy} = -\frac{4}{5}\alpha^2$

Note:

- The forces are expressed in Newton, the distances in meter.
- The distances are measured in the *y* direction, at an angle α to the vertical.
- It is assumed that the forces are applied at a point in the middle of the boat at ($x = 0$).
- The force directions are as indicated in the figure above.

Before you start your computations, answer the questions below.

3.1) From the perspective of numerical methods, what type of problem is this?

3.2) Name two numerical methods that can be used to solve this type of problem.

3.3) Discuss the advantages and disadvantages of these two methods.

3.4) Which numerical method would best be used to solve the problem listed above?

3.5) Now, solve the problem using the method of your choice. Find the solution with a relative accuracy of 0.001. Show your work. You can use the information that α is about 30 degrees.