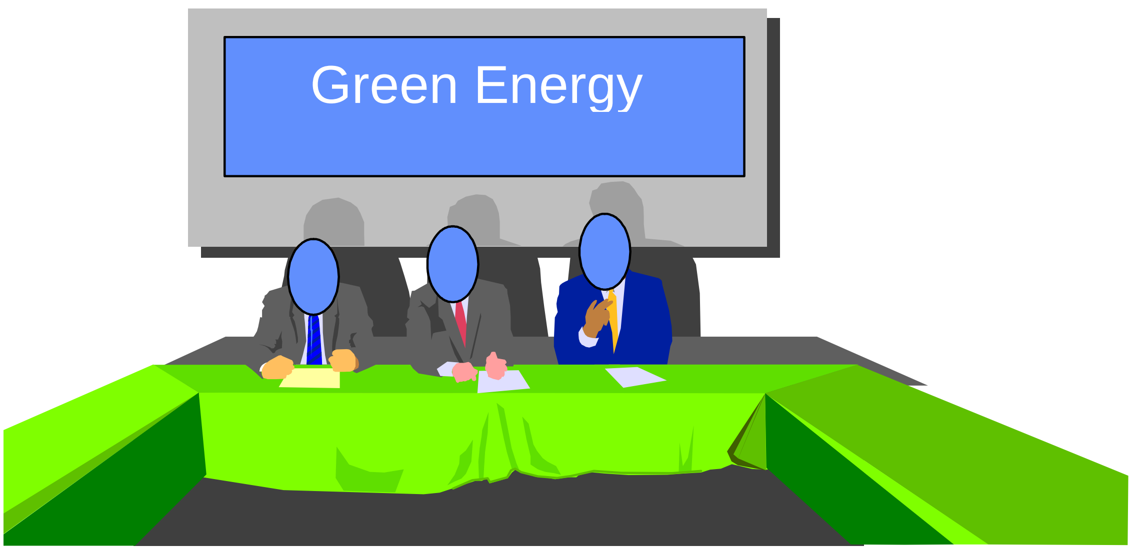
COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – FALL 2011

THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

S.-K. Choi, C. Paredis (Chair), D. Rosen, S. Sitaraman, and Y. Wang



- All questions in this exam have a common theme: ***Green Energy***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

1) Geometric Modeling

In this problem you will model part of the airfoil of a power generation wind turbine. Assume that the section of the blade shown below can be modeled using three Bezier curves, as shown in the plot below. The coordinates of the control vertices (CVs) are:

Curve A: <P0, P1, P2>

Curve B: <P2, P3, P4>

Curve C: <P4, P5, P6>

P0: (0,1)

P1: (*a*, *a*)

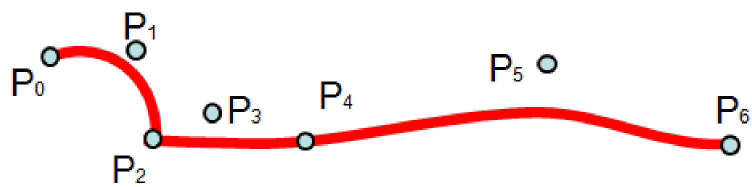
P2: (1, 0)

P3: (*b*, 0.4)

P4: (2, 0)

P5: (4,*c*)

P6: (6, 0.2)



1) Compute the equations for the three Bezier curves. Plug in the control points.

2) The curve A is a part of a circle. Find the value of *a* such that the Bezier curve defined by the points P0, P1, and P2 can best approximate a quarter circle of radius 1. What is the maximum deviation of this from a circle of radius 1?

3) Derive the condition for the combined curve to have G^1 parametric continuity at the joint point P4 in terms of P3, P4, and P5. If we move P3 to (1.5,0.4), what is the value of *c* for point P5 needed to satisfy the G^1 continuity?

4) Suppose that we join two Bezier curves using the control point sequences <P2, P4, P4> and <P4, P4, P6>, respectively. Does this composite curve have G^1 parametric continuity at P4? Provide detailed discussions.

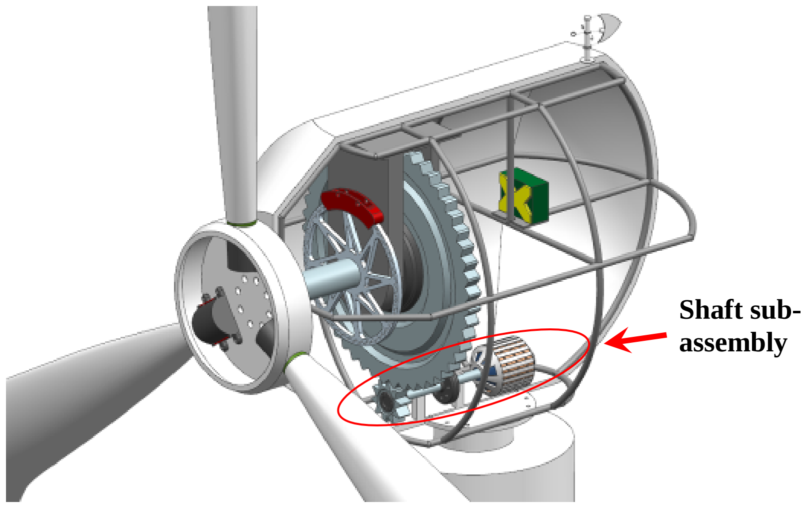
Bezier curve equations:

$$b(u) = \sum_{i=0}^{n} \underset{\sim}{p}_i B_{i,n}(u)$$

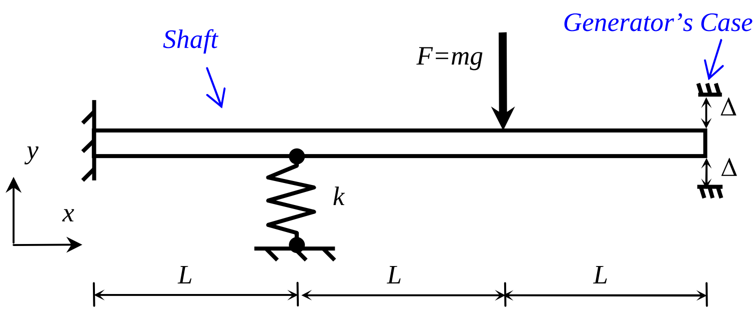
$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

2) Finite-Element Analysis

In designing a wind turbine as shown in the figure to the right, we need to analyze the shaft sub-assembly.



In the simplified model shown on the right, the shaft has a length of *3L*, a cross-section area of *A*, and a moment of inertia *I*. The shaft is made of a material with a modulus of elasticity *E*. The shaft is rigidly clamped on the left end and is free at the right end. The shaft is also supported by a spring with a stiffness of *k* at a distance *L* from the left end. For simplification purpose, a permanent magnet with a mass of *m* is assumed to be attached at a distance *L* from the right end of the shaft tip. There is a gap Δ between the right-end tip of the shaft and the generator's case.



You are asked to analyze the structure using finite-element formulation.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down the element stiffness matrix and assembly stiffness matrix.
5. Determine the vertical deflection of the shaft at the right-end tip, assuming that the tip does not touch the case. Show all steps to find results. No need to calculate the final values.
6. Now assume that with a larger magnet with a mass *M* attached in a second design and that *only* the right tip of the shaft touches the case. Determine the deflection of the shaft where the spring is attached. Show all steps to find results. No need to calculate the final values.

<u>Element A - Stiffness Matrix</u>

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where *E, A,* and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines.

<u>Element B - Stiffness Matrix</u>

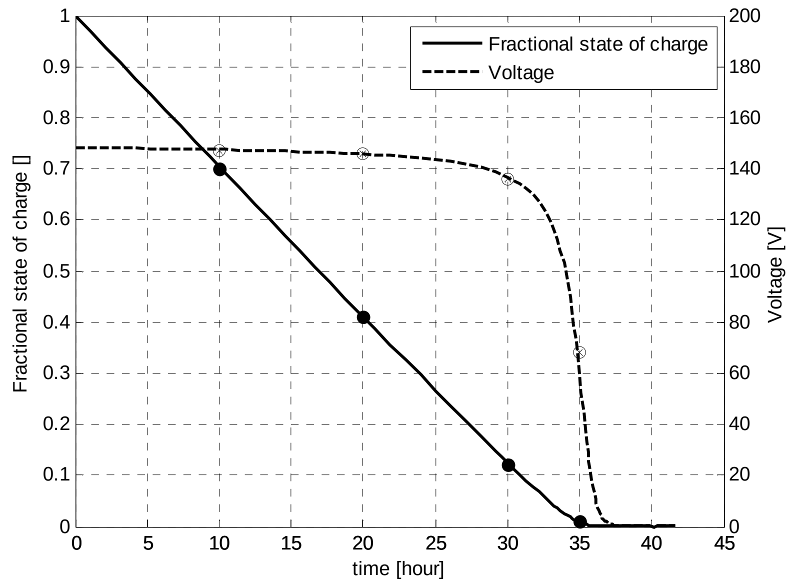
$$[K]=\frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E, I,* and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

3) Numerical Methods

To make electric vehicles commercially viable, a significant improvement in battery technology is still needed. To move towards that goal, you are testing and characterizing the behavior of a new battery. After fully charging the battery, you discharge it slowly by connecting a 100 Ohm resistor between its two terminals. You keep track of the fractional state of charge of the battery by continually measuring the current flowing through the resistor. After 10, 20, 30 and 35 hours, you also measure the voltage of the battery. All measurements are made with a relative accuracy of $\pm 3\%$. You obtain the following data:

Time, t [hour]	Fractional state of charge, $fsoc$	Voltage, v [V]
10	70%	147
20	41%	146
30	12%	136
35	1%	68



Example test result — the fractional state of charge and the voltage decrease over time

The equations that govern this experimental setup (the Shepherd equation and Ohm's Law) are:

$$v = V_{nom} - Ri - \frac{Ki}{fsoc}$$

$$v = R_{load} i$$

with $V_{nom} = 150V$ and $R_{load} = 100\Omega$. Your task is to determine the battery characteristics, R and K.

Questions:

a) First, derive the equations for computing R and K.

b) What are the numerical assumptions you have made in your derivation? Are these assumptions reasonable?

c) Compute the values for R and K.

d) How could you obtain more accurate estimates of R and K.