M.E. Ph.D. Qualifier Exam
Spring Semester 2003

GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2003

Computer-Aided Engineering
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

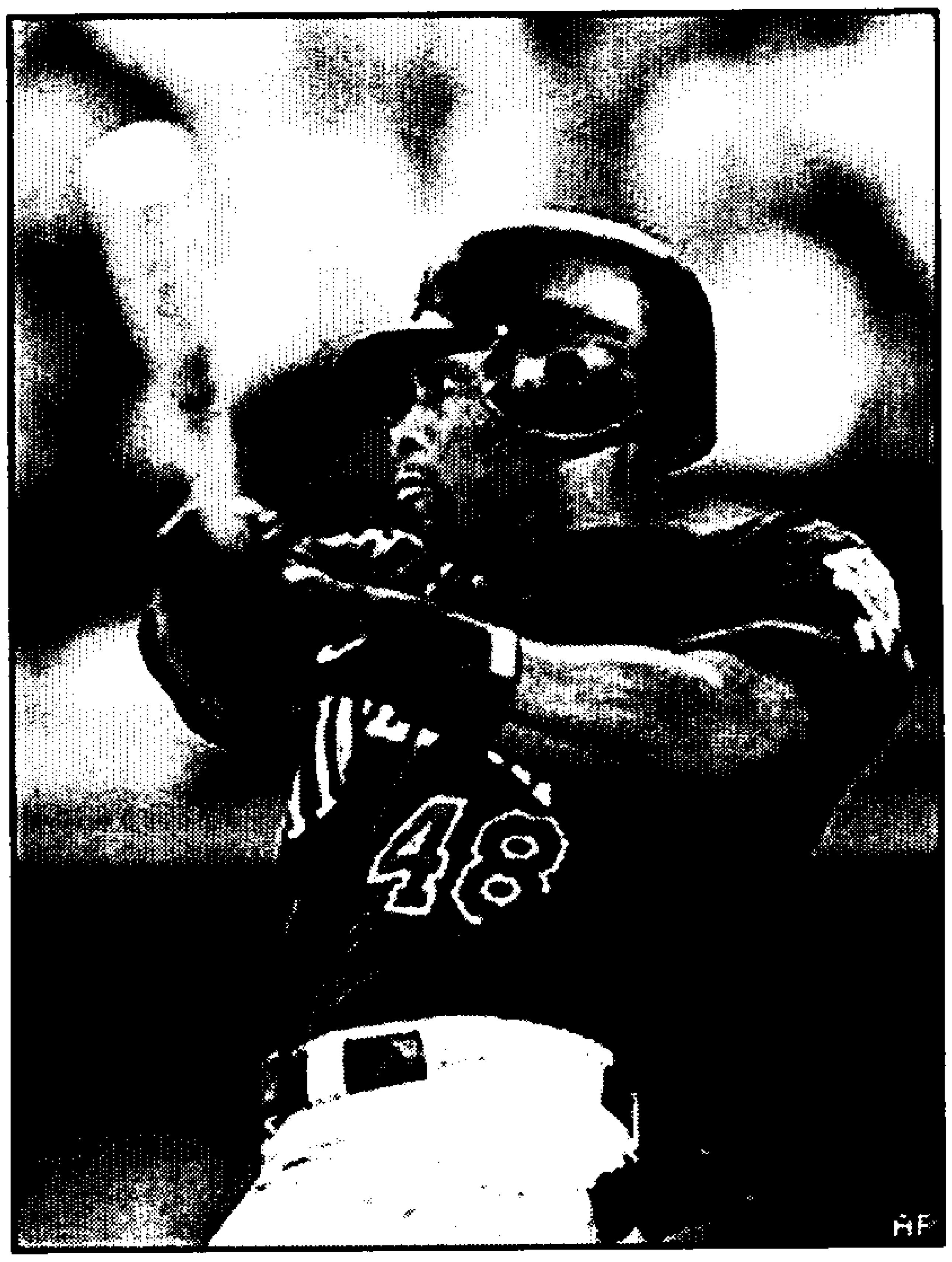
Answer **3 of the 4** questions.

COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Spring 2003

THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

Bras, Fulton, Rosen (Chair), and Sitaraman



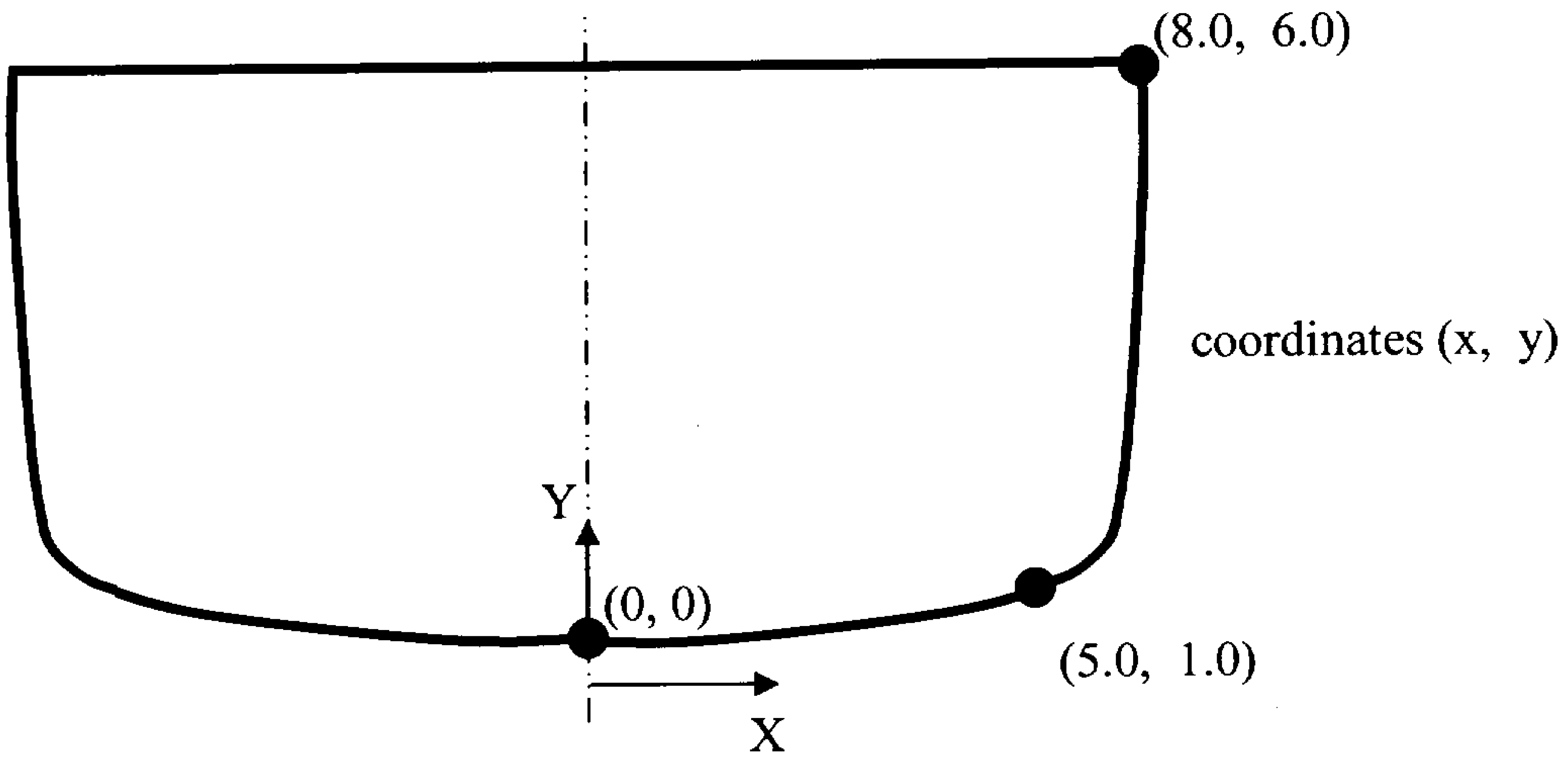
- All questions in this exam have a common theme: ***Baseball***
- Answer **3 out of the 4** questions. Indicate which questions you want graded.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Answer **3 of the 4** questions.

Question 1
Geometric Modeling of Batting Cage Frame

We will investigate the shape of a batting cage frame using the sketch below. It shows a typical transverse cross-section through the cage (z axis is vertical). The shape is symmetric about the y axis. To save time and simplify the construction of complex shapes, designers carry out the design of one side of the frame and then mirror the other side to form the entire frame. The questions for this problem are related to the geometric modeling of the frame.



Dimensions are in feet.

a First, sketch the above cage shape. On the sketch, indicate how you would use a composite cubic Bezier curve to model the shape of the right-hand side of the frame. Use 3 cubic Bezier segments in your composite Bezier curve. Make sure the resultant composite curve passes through the 3 indicated points. Draw in the nodal vertices and related polygons to define the Bezier segments. Label points where curves join.

b Describe how you modeled continuity between curve segments. How did you choose the location of the start/end points of each curve? What is the level of continuity?

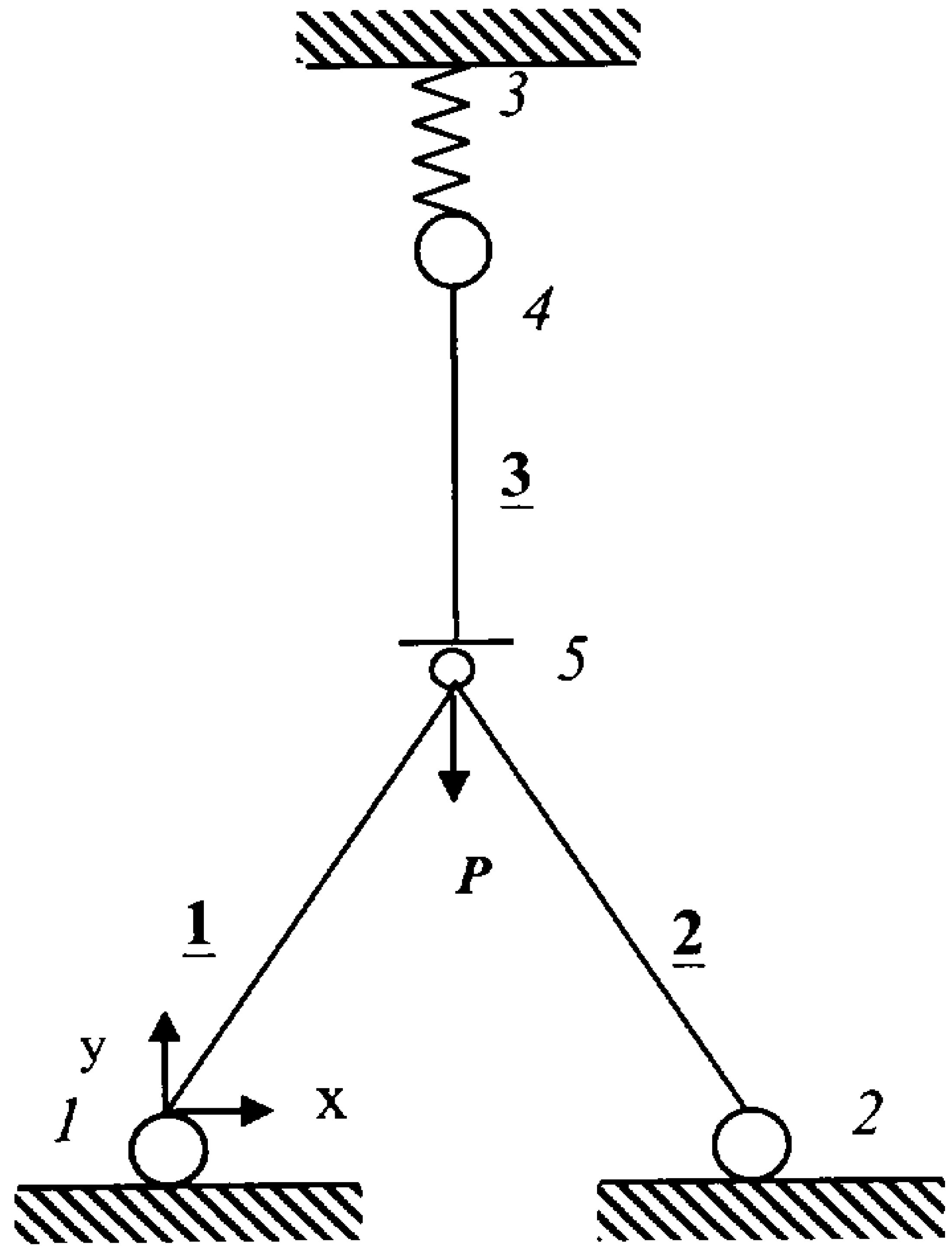
c Using the indicated vertex coordinates, compute the coordinates for the corresponding points on the cage's left side.

d Discuss the advantages and disadvantages of using cubic Bezier curves and cubic splines for modeling this cage shape. What type of curve seems most appropriate and why?

Answer **3 of the 4** questions.

Question 2
Finite Element Modeling of Stadium Structure

The figure below shows a structure in the baseball stadium consisting of three members. The **bold underlined** numbers indicate member numbers and the *italicized* numbers indicate joint numbers. Joints 1, 2, 4, and 5 are pin joints and are represented as circles. A vertical spring with a spring constant k connects joints 3 and 4 as shown in the figure. Joint 5 is a pin joint between members 1 and 2. Member 3 has a flat bottom surface that touches the pin joint at 5.



All members are of equal length L, cross-section area A, and made of the same material with a modulus of elasticity E. The distance between joints 1 and 2 is L. A vertical force of magnitude P acts at joint 5 as shown in the figure.

Use Finite-Element Formulation to analyze the structure.

- Develop individual stiffness matrix for each member
- Assemble and show the assembly stiffness matrix; use symmetry wherever possible
- Show the assembled equilibrium equation in matrix form
- Identify the boundary conditions
- Show how you will solve the displacement at node 5
- Suppose the load P acts horizontally in the positive X-axis direction, instead of vertically downwards, discuss how you will do the analysis.

Element Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where E, A, and L are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; l and m are direction cosines of the element with respect to X and Y axes and are given by:

$$l=\frac{x_2-x_1}{L}$$

$$m=\frac{y_2-y_1}{L}$$

Answer **3 of the 4** questions.

Question 3
Baseball and Bat Collision

You are now working for the Atlanta Braves baseball team who want you to tell them more about the physics of baseball – bat collisions.

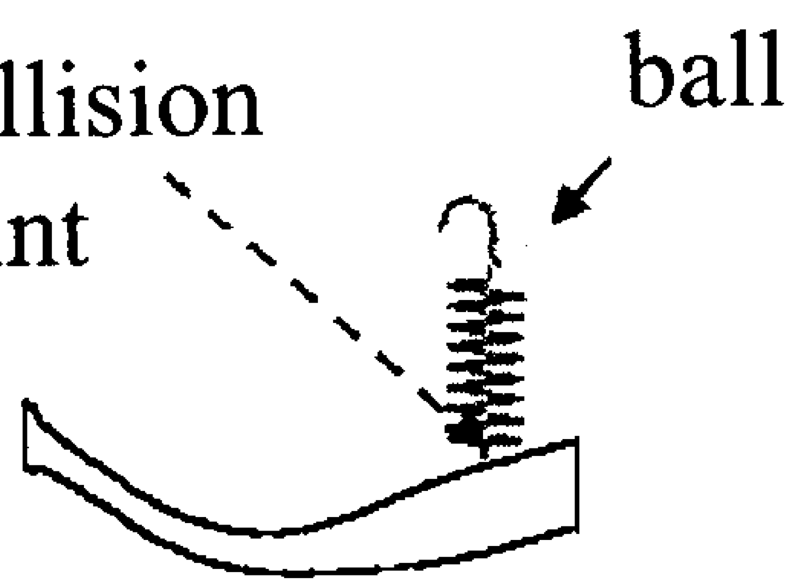


Figure 1 - Baseball and bat collision and deformation

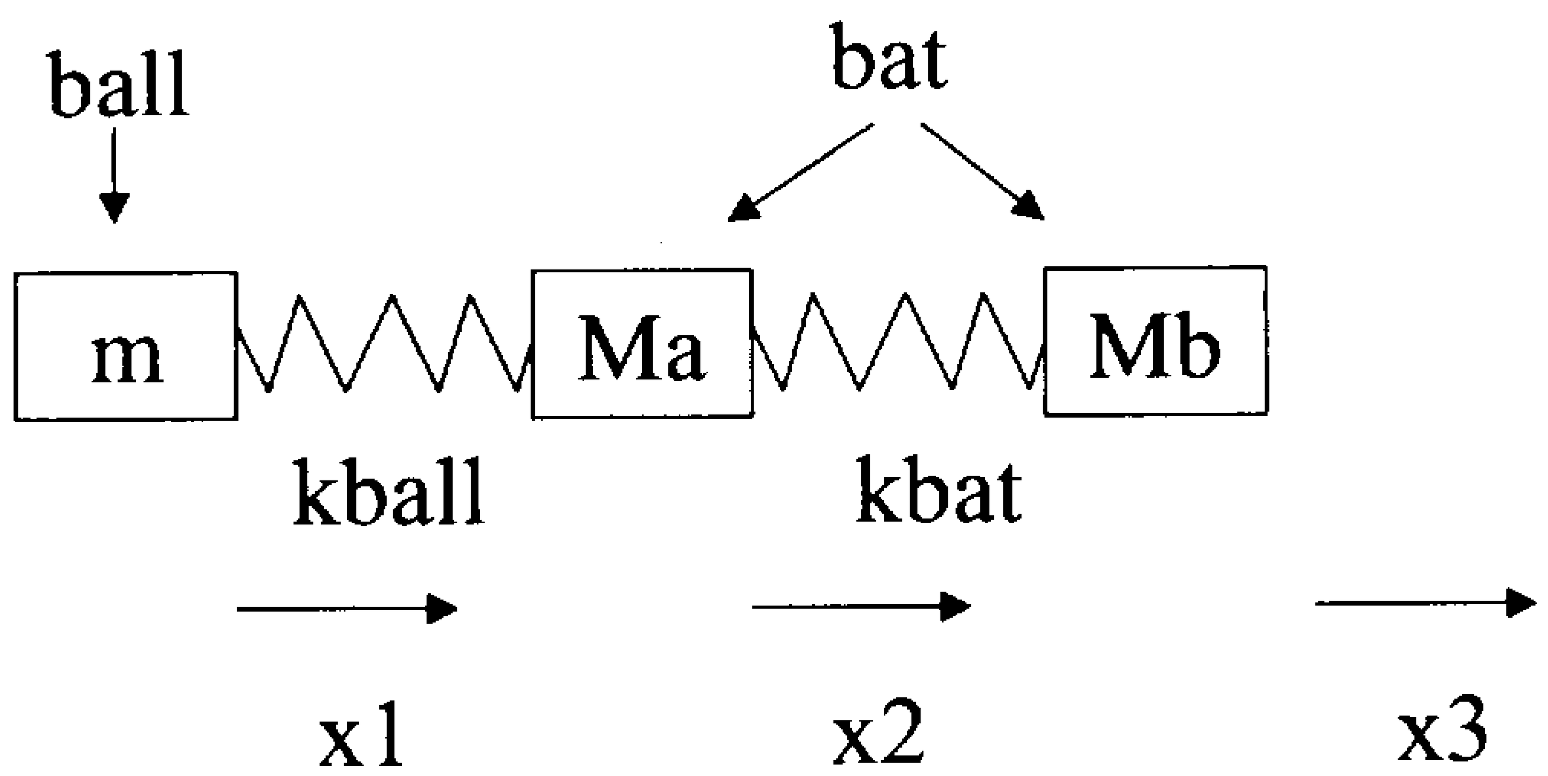


Figure 2 - Baseball and bat collision and deformation modeled as a mass-spring system

One of your references from which the above figures are taken is the article "Dynamics of the baseball–bat collision" by Alan M. Nathan, Am. J. Phys. 68, 979 (2000). Figure 1 represents schematically what happens if a baseball hits the bat. Both the bat and baseball deform. In Figure 2, the collision between the baseball and the bat is represented as a mass-spring system. This is a one-dimensional problem, with the "bat" initially at rest and the ball initially with speed vi. The ball impacts the mass Ma and the ultimate goal is to find the rebound speed of the ball and the energy going into recoil and vibrations of the bat after the ball and bat separate.

Assume m = 1, Ma = 2, and Mb = 4 units. Also assume that kball = 1 and kbat = 50 (kball/kbat = 0.02 for wooden bats).

Make any assumptions you think are necessary to answer the following questions.

a) Set up the equations of motions for the system shown in Figure 2

b) Find the largest eigenvalue using Powers method for this system.

c) Name three other methods for finding eigenvalues and discuss their advantages and disadvantages.

Answer **3 of the 4** questions.

Question 4

Geometric Modeling + Numerical Methods

Consider a batter swinging his bat at a pitch. What are the mathematical conditions for hitting the ball? Well, the paths taken by the bat and the ball must intersect spatially and temporally. Let's model the problem in this manner.

- First, assume that the path taken by the ball is a cubic Bezier curve, where the parameter is t, time (appropriately normalized so that the entire trajectory occurs from t = 0 to t = 1). This should be a 3-D curve, **b**(t).
- Second, assume that we are only interested in the front profile curve of the bat. And, this profile curve sweeps out a surface patch that can be modeled by a bicubic Bezier surface patch, **p**(u,t). One parameter of this patch is u (along the bat's front profile), while the other is time, t. The same parameter t is used for both the bat's trajectory and the ball's.

Thus, the batter hits the ball if p(u,t) and b(t) intersect. Your challenge in this problem is to develop a method for determining if the batter hits the ball. See the schematic below.

Relevant equations include:

Ball: $\mathbf{b}(t) = \sum_{i=0}^{3} \mathbf{p}_i B_{i,3}(t)$ Bat: $\mathbf{p}(u,t) = \sum_{j=0}^{3}\sum_{i=0}^{3} \mathbf{p}_{i,j} B_{i,3}(u) B_{j,3}(t)$

where the $B_{i,3}(t)$, etc., are the Bezier blending functions.

a. Treat the bat's trajectory as a typical spatial surface patch (i.e., replace parameter t with w). Now you have three parameters among the bat's surface and ball's trajectory curve. Specify the mathematical conditions for the intersection of the surface and the curve.

b. Propose an algorithm for solving this system of equations to determine the spatial intersection of a surface and curve. Make use of the numerical methods that you are familiar with.

c. What is the mathematical condition for the bat hitting the ball (just because the surface and curve intersect spatially, that does not mean that the bat actually hits the ball)?