COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Spring 2006

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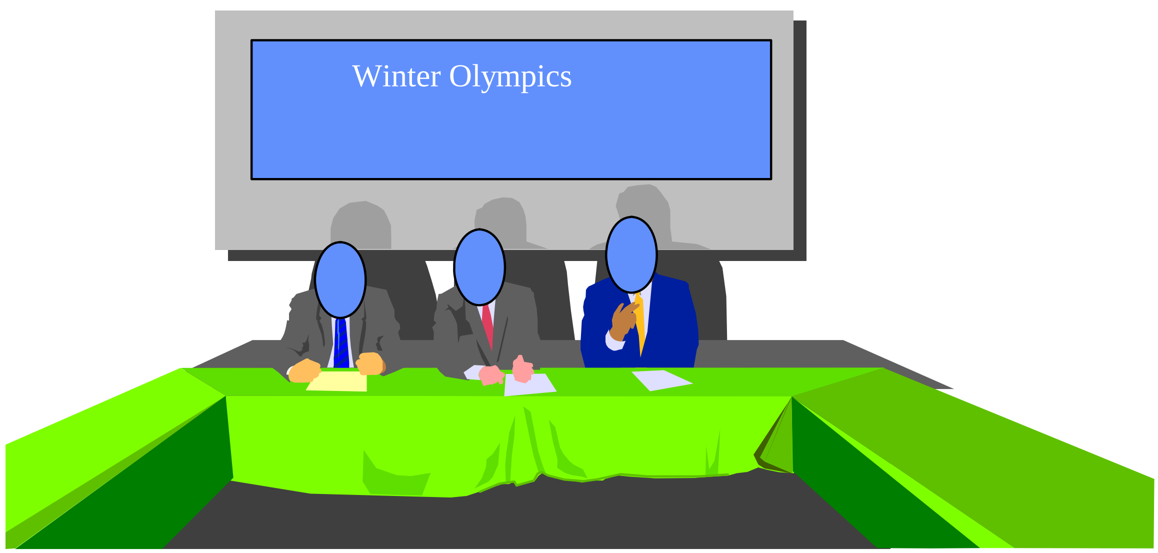


- All questions in this exam have a common theme: ***Winter Olympics***

- Answer all questions.

- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.

- Show all steps and calculations.

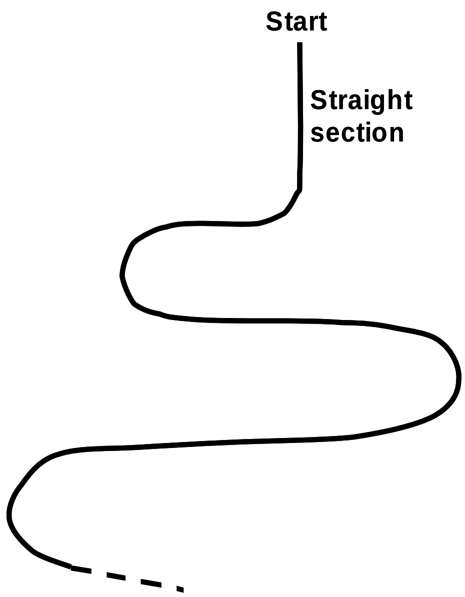
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1 - Geometric Modeling

The bobsled and luge are very exciting Winter Olympics sports. Riders can reach speeds over 70 mph and turns can cause riders to experience high ‘g’ loading. In this problem, you will model a simplified bobsled track.

Bobsled tracks are complex 3D surfaces. For our purposes, we will project the 3D surface into 2D and just treat the track as a curve. Consider the simplified 2D curve model of the track shown below. This is the start of the track and the first few curves.



Bezier curve equations:

$$b(u) = \sum_{i=0}^{n} \underset{\sim}{p}_i B_{i,n}(u)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

a) Sketch the curve on your answer sheet. Develop a model of the curve as a 2D composite, cubic Bezier curve. Ensure G1 continuity between curve segments. Explain how you ensure G1 continuity. Show the curves with control vertices.

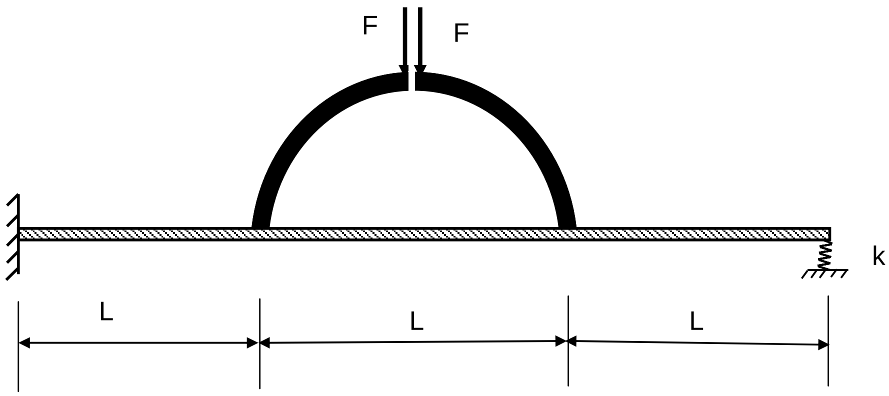
b) Due to the riders’ high speeds, the track designers need to improve the continuity conditions. Sketch the curve again and develop a model of it as a 4th degree composite Bezier curve with G2 continuity among curve segments. Recall that G2 continuity can be achieved with 5 collinear control vertices (2D curves). Illustrate how you achieve G2 continuity on your sketch.

c) Assume that a 4th degree Bezier curve has the following control vertices: (12,6), (12,2), (0,8), (0,5), (0,0). Compute the 2D (X, Y) coordinates of the point at u = 0.5 on this curve.

d) On a separate sketch, show an example of G2 continuity between two cubic Bezier curves. Explain how you ensure G2 continuity.

e) Show an example of G2 continuity among three cubic Bezier curves. What can you say about the shape modeling capabilities of composite cubic Bezier curves with G2 continuity?

Question 2 – Finite Element Analysis



A long structure in one of the Olympic stadiums can be simplified as shown in the figure. The structure has a length of *3L* and a square cross-section with a side of *a*. The structure is made of a material with a modulus of elasticity *E*. The structure is rigidly clamped on one end and is supported by a spring with a stiffness of *k* at the other end. A rigid semicircular member with a diameter *L* is attached on the long structure as shown in figure. The rigid semicircular member is slit at its top center and a downward acting force of magnitude F is applied to each quarter circle as shown in the figure. Assume that the two quarter circles do not touch each other during deformation. You are asked to determine the vertical deflection of the long structure.

1. Use an appropriate finite-element formulation to solve the problem.
2. State all of your assumptions clearly.
3. Show all of your calculations.
4. Show the boundary conditions and loading conditions.
5. Write down element stiffness matrix and assembly stiffness matrix.
6. Determine the vertical deflection of the long structure at locations *L* and *2L* from the clamped end. Show all appropriate steps. You need not solve.

Element A - Stiffness Matrix

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l = \frac{x_2 - x_1}{L} \qquad m = \frac{y_2 - y_1}{L}$$

where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines of the element with respect to *X* and *Y* axes.

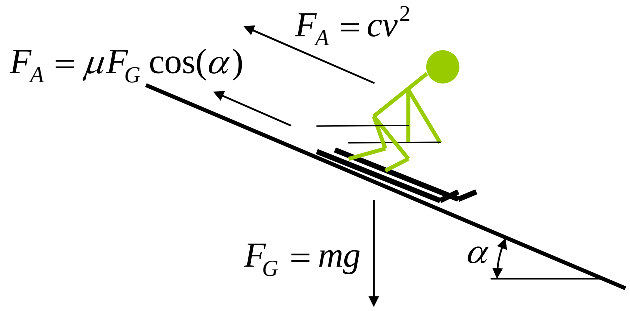
Element B - Stiffness Matrix

$$[K] = \frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E*, *I*, and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Methods

To prepare for the Olympics in 2010, the folks in Vancouver are designing a new Nordic Ski Jump site. To make sure the jumping hill is safe, you are asked to analyze the speed the jumpers will achieve at the bottom of the hill.



Questions:

a) Formulate a set of differential equations that describe the motion of the ski jumper while sliding down the straight-line section of the ramp. (see figure above). The jumper experiences three forces: gravitation, friction between skis and snow, and aerodynamic drag. Use the following parameters:

- the slope, α : 30 degrees
- the mass of the skier, *m*: 80 kg
- the drag constant, *c*: 0.4 kg/m
- the friction coefficient, μ : 0.1 [dimensionless]
- the gravitational acceleration, *g*: use *g* = 10 m/s^2

b) Use the mid-point method to compute the position of the ski jumper after 1 seconds. Use the following parameters for solving the initial value problem:

- initial position = 0 m
- initial velocity = 1 m/s
- integration step size = 0.5 second

The equations for the mid-point method are:

$$y_{i+1} = y_i + \phi h$$
$$k_1 = f(t_i, y_i)$$
$$\phi = f(t_i + \frac{h}{2}, y_i + k_1 \frac{h}{2})$$

c) Is the mid-point method a good method for solving this problem? If not, which method would be better to use? In general, based on which characteristics would you decide on the appropriate algorithm for solving an initial value problem?