COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Spring 2007

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- All questions in this exam have a common theme: ***Storms and Snow Storms***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1:
Geometric Modeling

In this problem you will model the shape of a snowplow. Consider the "wing plow" on the front of the truck shown to the right. We are interested in models of the plow's front surface.

Assume that the plow surface can be modeled using linear lofted surface patches.

a) Derive the equation of a linear lofted surface between two quadratic Bezier curves.

b) Assume that the control vertices for the left quadratic Bezier curve are: (40,20,0), (-20, -20, 80), (0,0,120). Assume that the control vertices for the right quadratic Bezier curve are: (40,300,0), (10, 300, 40), (30,300,60). Plug these control vertices into your linear lofted surface equation and simply the x(u,w), y(u,w), z(u,w) equations.

c) Compute the point on the surface at u = w = 0.5. Does your answer make sense?

d) Sketch the surface in the YZ and XZ planes. I suggest plotting the control vertices first.

e) Assume that the left Bezier curve is not shaped properly. The designer decides to try a cubic Bezier curve on the left side, but does not want to change the right quadratic Bezier curve. Is it possible mathematically to derive a linear lofted surface patch between quadratic and cubic curves? If so, derive the surface equation. If not, explain why not (using mathematics if possible).

f) Trucks can control the position and orientation of their plows. We model these position / orientation changes using rigid-body transformations. Derive the homogeneous transformation matrix for a combination of a translation of (-5, 20, 50) followed by a rotation of –30 degrees about the Z axis. Set up the equation for transforming the control vertices from part (b) so that your surface model is transformed. You do not need to compute the transformed control vertex coordinates.

Bezier curve equations:

$$b(u) = \sum_{i=0}^{n} \underset{\sim}{p}_i B_{i,n}(u)$$

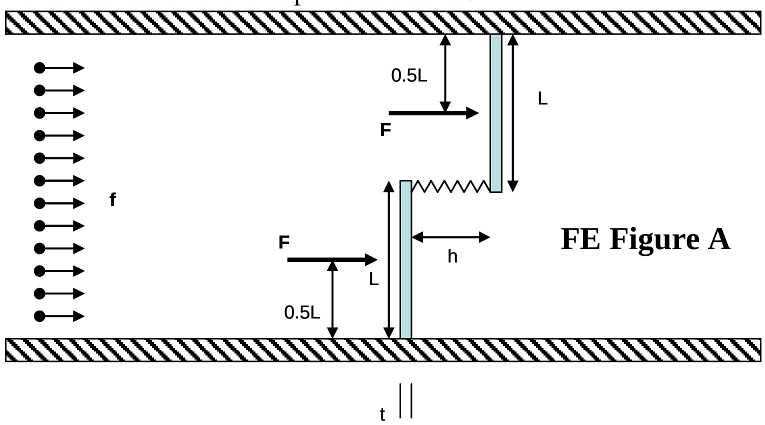
$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

Question 2:
Finite Element Analysis

The figure below (*FE Figure A*) shows a passage of width 2L. An engineer is asked to design a blocking structure to reduce potential damage to downstream entities due to various storms. The engineer has proposed two flap-like structures connected by a spring of stiffness *k* and length *h*, as shown in the figure. The flap-like structures are identical with a length of L and a thickness of *t*. Assume the out-of-plane depth of the flap-like structures to be *d*. The material used has a modulus of elasticity *E*.

Assume that the distributed storm force *f* across the passage can be modeled as two point forces of magnitude *F*, applied at half way on the flap-like structures as illustrated. Using an appropriate finite-element formulation, determine the horizontal deflection of the flap-like structures.

- State all assumptions clearly.
- Show the boundary conditions and loading conditions.
- Write down element stiffness matrix and assembly stiffness matrix.
- Determine the horizontal deflection at the tip of the flap-like structures.
- Determine the change in the spring length.



Element M - Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

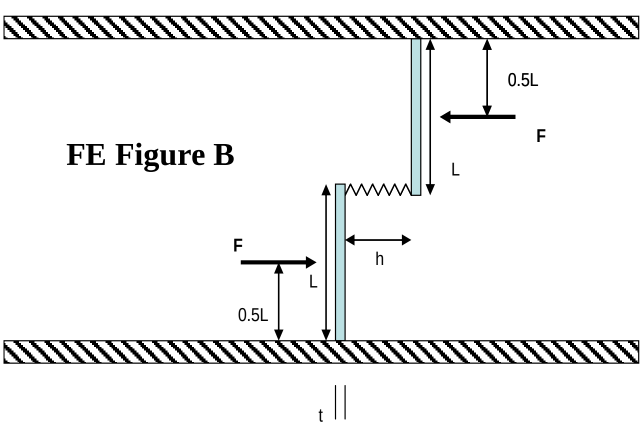
$$l=\frac{x_2-x_1}{L} \qquad m=\frac{y_2-y_1}{L}$$

where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines of the element with respect to *X* and *Y* axes.

Element N - Stiffness Matrix

$$[K]=\frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E*, *I*, and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;



Assume now that the concentrated force *F* is applied on the opposite sides of the flaps as illustrated in *FE Figure B*. Using finite elements, show how you will formulate this problem to determine the deflection of the flap-like structures and the spring deformation. You need not solve the problem. It is sufficient to show the assembly matrix and appropriate boundary and loading conditions.

Problem 3:
Numerical Methods

In a heavy winter storm, it is not uncommon for roofs to collapse under the heavy weight of accumulated snow. During a recent storm, we have measured to following snow-fall densities and rates:

Time	10PM	11PM	12AM	1AM	3AM	5AM	6AM
Density [kg/m^3]	200	190	180	160	140	145	150
Rate [m/hour]	0.022	0.030	0.025	0.014	0.032	0.024	0.02

Questions:
a) What is the mass of the snow that accumulated between the times of 10PM and 8AM on a roof with a surface area of 100m^2? Compute your answer as accurately as possible. Show your work. To aid your memory we have included some equations below.

b) Estimate the error in your computation. Which assumptions did you make in your error computations?

c) If the snow density and fall rate were given to you as analytical functions that you could evaluate for any arbitrary time instant, which method would you use to compute the mass of the snow? Justify your answer.

Equations:

$$I = \int_a^b f(x)dx \cong \int_a^b f_n(x)dx$$

$$I \cong (b-a)\frac{f(a)+f(b)}{2} \qquad E_t = -\frac{(b-a)^3}{12} f''(\xi)$$

$$I \cong (b-a)\frac{f(x_0)+2\sum_{i=1}^{n-1} f(x_i)+f(x_n)}{2n} \qquad E_t = -\frac{(b-a)^3}{12n^2}\bar{f}''$$

$$I \cong (b-a)\frac{f(x_0)+4f(x_1)+f(x_2)}{6} \qquad E_t = -\frac{(b-a)^5}{2880} f^{(4)}(\xi)$$

$$I \cong (b-a)\frac{f(x_0)+4\sum_{i=1,3,5}^{n-1} f(x_i)+2\sum_{j=2,4,6}^{n-2} f(x_j)+f(x_n)}{3n} \qquad E_t = -\frac{(b-a)^5}{180n^4}\bar{f}^{(4)}$$

$$I \cong (b-a)\frac{f(x_0)+3f(x_1)+3f(x_2)+f(x_3)}{8} \qquad E_t = -\frac{(b-a)^5}{6480} f^{(4)}(\xi)$$