COMPUTER-AIDED ENGINEERING
Ph.D. QUALIFIER EXAM – Spring 2010

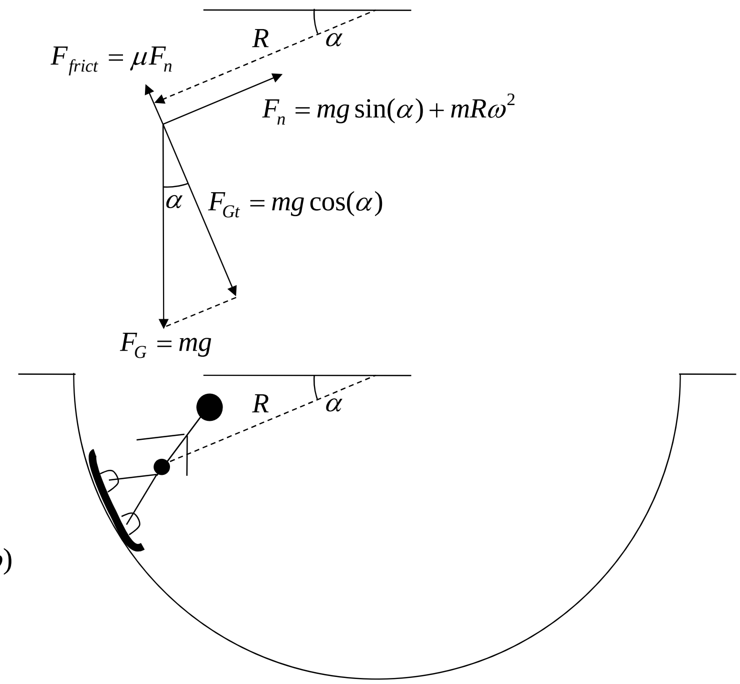
THE GEORGE W. WOODRUFF
SCHOOL OF MECHANICAL ENGINEERING
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

S.K. Choi, C. Paredis, D. Rosen (Chair), S. Sitaraman, and Y. Wang (observer)

- All questions have a common theme: ***Winter Olympics***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

GOOD LUCK!

Question 1: Numerical Methods



In his preparation for another gold medal in snowboarding, Shawn White has spared no effort. He has been practicing on his own private half-pipe course, which comes equipped with a foam-filled pit for guaranteed soft landings. In addition, he has contacted you with the request to perform a careful analysis of the dynamics of a snowboarder as he reenters the half-pipe after a jump.
You recognize that the motion of the snowboarder can be approximately modeled by a differential equation similar to the one for a pendulum:

$$mR^2 \frac{d\omega}{dt} = mg\cos(\alpha) - \mu(mg\sin(\alpha) + mR\omega^2)\,\mathrm{sgn}(\omega)$$

with rotational speed, $\omega = d\alpha / dt$, and the following values:
$m = 80kg$, $\mu = 0.1$, $g = 10m/s^2$, $R = 5m$.
(Note: sgn(x) is the signum function which equals 1 if $x>0$, -1 if $x<0$, and 0 if $x=0$).

Questions:
a) Use Heun's method to compute the position of the snowboarder after 1 second. Use the following parameters for solving the initial value problem:

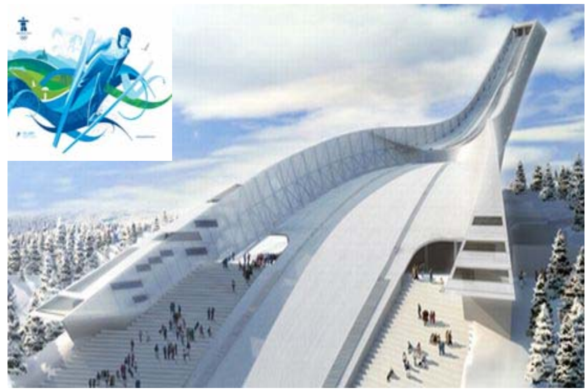
- initial angle = 0 deg
  (i.e., just entering the semi-circle with the snowboard pointing straight down)
- initial velocity = 5 m/s (straight down)
- integration step size = 0.5 second

The equations for Heun's method are:

$$y_{i+1} = y_i + \phi h$$
$$k_1 = f(t_i, y_i)$$
$$k_2 = f(t_i + h, y_i + k_1 h)$$
$$\phi = \frac{k_1 + k_2}{2}$$

b) With an integration step size of h=0.5 seconds, the true global error for α after 1 second is: $|E_t| = 7e-4$. How large would the error have been had you used a step size of h=0.1 seconds? Explain. How large would the error have been had you used a 4th order Runge-Kutta method with step-size of h=0.5 seconds?

Question 2: Geometric Modeling



In this problem you will model part of the slope of a ski jump tower.

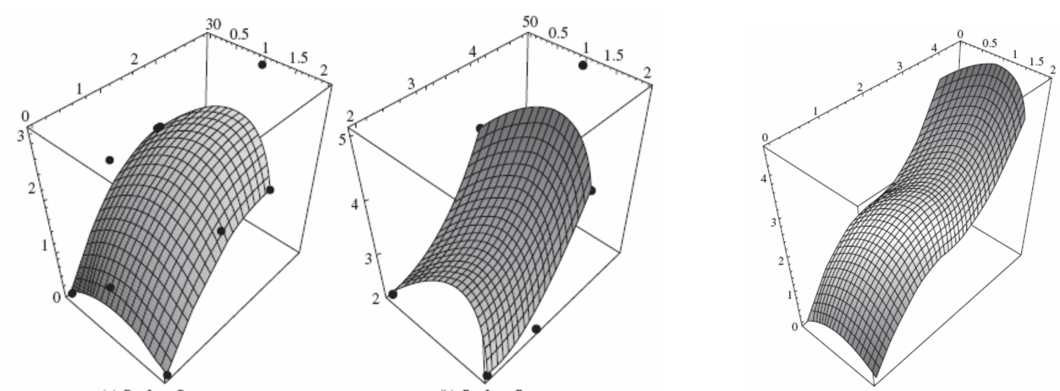
Assume that the section of the slope shown below can be modeled as blending two quadratic Bezier patches. The control vertices of this patch are:

Surface A	$\mathbf{p}_{i0}$	$\mathbf{p}_{i1}$	$\mathbf{p}_{i2}$
$\mathbf{p}_{0j}$	0,0,0	1,1,0	2,0,0
$\mathbf{p}_{1j}$	0,1,1	1,2,1	2,1,1
$\mathbf{p}_{2j}$	0,0,2	1,1,2	2,0,2
Surface B	$\mathbf{r}_{i0}$	$\mathbf{r}_{i1}$	$\mathbf{r}_{i2}$
$\mathbf{r}_{0j}$	0,0,2	1,1,2	2,0,2
$\mathbf{r}_{1j}$	r_{10}	r_{11}	r_{12}
$\mathbf{r}_{2j}$	0,4,4	1,5,5	2,4,4

Bezier surface equations:

$$p(u,w) = \sum_{i=0}^{n}\sum_{j=0}^{m} \underset{\sim}{p}_{ij} B_{i,n}(u) B_{j,m}(w)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$



(a) Surface A (b) Surface B (c) Composite Bezier Surface

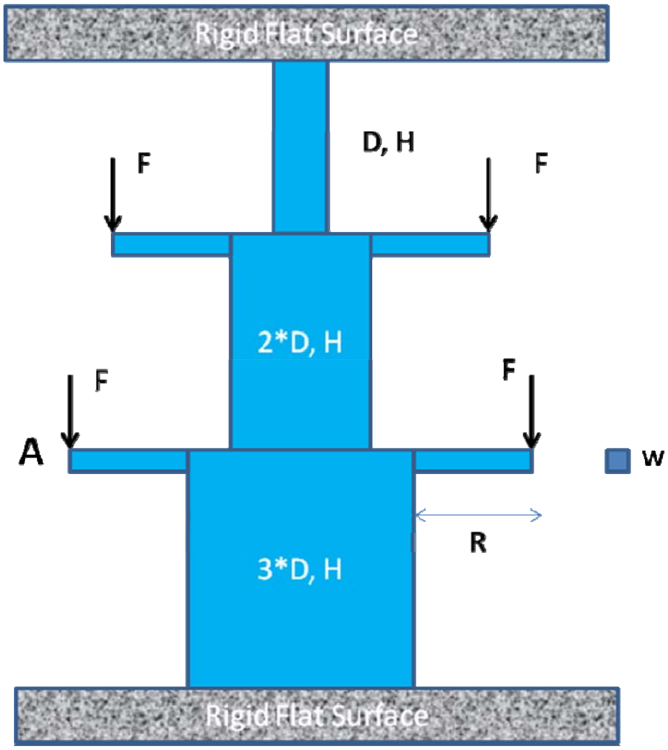
a) Derive the equation of the Bezier patch – expand the equation for $p(u,w)$ below so that your answer is in terms of the blending functions polynomials in u and w.

b) Now, derive the matrix form of the surface equations.

c) Compute the point and the unit normal to the Surface A at $u = 0$, $w = 0$.

d) Assume that G^1 continuity is desired between surface patches which means the tangent plane of patch A at $u = 1$ must coincide with that of patch B at $u = 0$ for $w \in [0,1]$. Determine the unknown control points, r_{10}, r_{11}, and r_{12}.

Problem 3: Finite Element Analysis



A totem pole like structure in one of the Olympic venues is constrained between two rigid flat surfaces, as shown in the figure. The top part of the structure is a cylinder with a diameter *D* and a height *H*, the middle part is a cylinder with a diameter *2*D* and a height *H*, and the bottom part is a cylinder with a diameter *3*D* and a height *H*. There are overhangs that are rigidly attached to diametrically opposite sides of the cylinders as shown in the figure. The overhangs have a length *R* and have a square cross-section with a side *w*. Force *F* is applied at the tip of each overhang. The entire structure is made of the same material with a modulus of elasticity *E*.

1. Using appropriate finite-element formulation, determine how much *F* will move at location *A*
2. State all of your assumptions clearly.
3. Show all of your calculations.
4. Write down necessary stiffness matrices.
5. Show the boundary conditions and loading conditions. Solve for movement at *A*.

Element A - Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \qquad l=\frac{x_2-x_1}{L} \qquad m=\frac{y_2-y_1}{L}$$

$$[K]=\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines of the element with respect to *X* and *Y* axes.

Element B - Stiffness Matrix

$$[K]=\frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E*, *I*, and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;