COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – SPRING 2012

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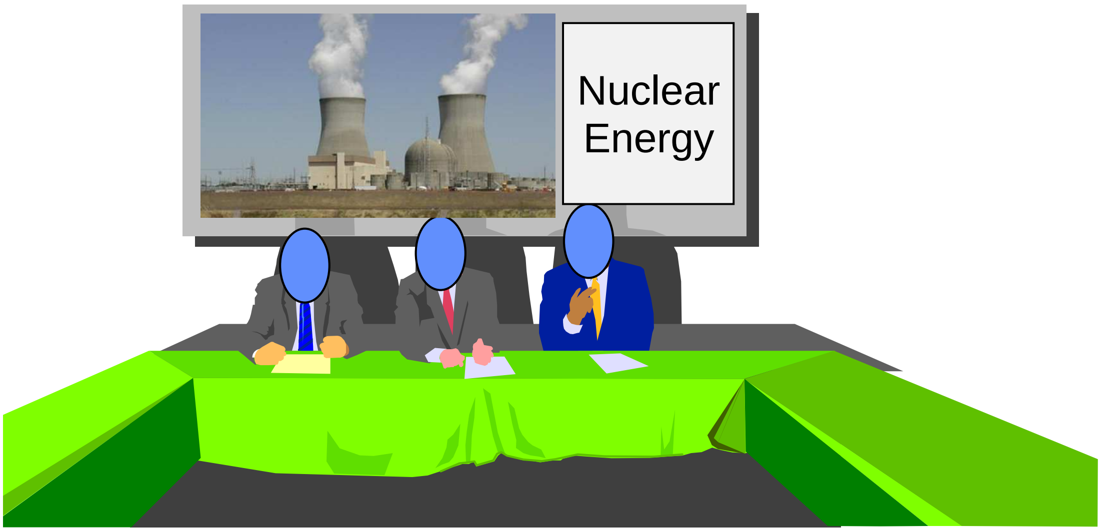


- All questions in this exam have a common theme: ***Nuclear Energy Plants***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

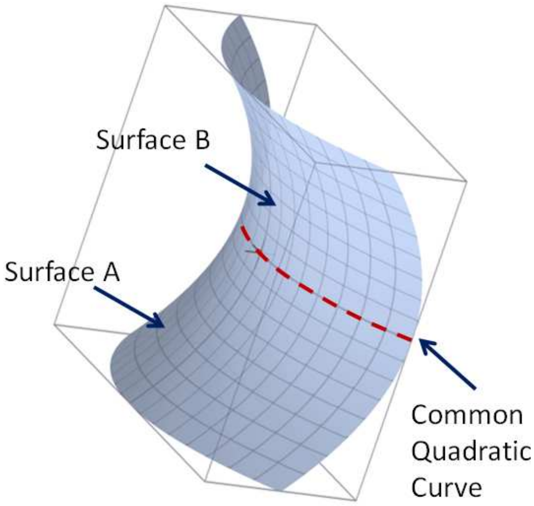
GOOD LUCK!

1) Geometric Modeling

In this problem you will model part of the nuclear plant cooling tower.

Assume that the section of the tower shown below can be modeled by blending two quadratic Bezier patches. The control vertices of these patches are:

Surface A	$\mathbf{p}_{i0}$	$\mathbf{p}_{i1}$	$\mathbf{p}_{i2}$
$\mathbf{p}_{0j}$	0,0,0	0,1,2	0,2,2
$\mathbf{p}_{1j}$	0,0,1	1,1,3	1,3,3
$\mathbf{p}_{2j}$	2,0,0	2,1,2	2,2,2
Surface B	$\mathbf{r}_{i0}$	$\mathbf{r}_{i1}$	$\mathbf{r}_{i2}$
$\mathbf{r}_{0j}$	0,2,2	r_{01}	0,4,4
$\mathbf{r}_{1j}$	1,3,3	r_{11}	1,5,5
$\mathbf{r}_{2j}$	2,2,2	r_{21}	2,4,4



The composite surface is joined at the common quadratic curve: $\{p_{02}, p_{12}, p_{22}\} = \{r_{00}, r_{10}, r_{20}\}$.

Bezier surface equations are

$$p(u,w) = \sum_{i=0}^{n} \sum_{j=0}^{m} \underset{\sim}{p}_{ij} B_{i,n}(u) B_{j,m}(w)$$

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

a) Derive the equation of Surface A in matrix form.

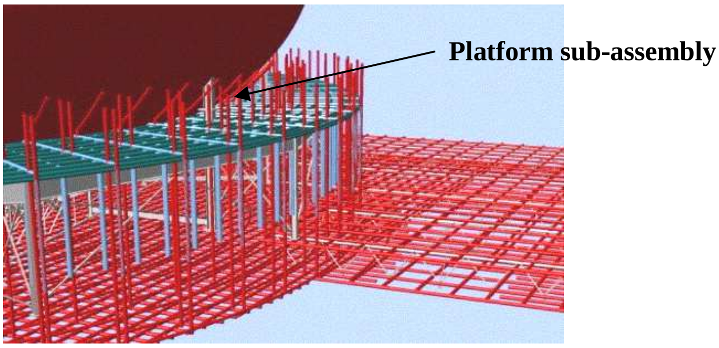
b) Now, let the Bezier patch $p(u,w)=[p_x,\ p_y,\ p_z]$; expand the z component (p_z) only in terms of the blending functions polynomials in u and w.

c) G^1 continuity is desired between surface patches. Determine the unknown control points, r_{01}, r_{11}, and r_{21}.

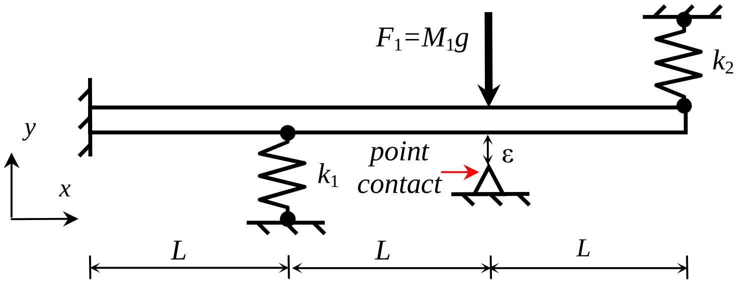
d) Assume that four corner points of a patch are given and we know that the first partial derivatives and mixed (twist) partial derivatives at each corner. Is it possible to construct a <u>bicubic</u> Bezier patch? Explain why or why not.

2) Finite-Element Analysis

In designing the AP1000 power plant with the containment support structure as shown in the figure to the left, we need to analyze the platform sub-assembly.



In the simplified model shown on the right, the horizontal structure has a length of *3L*, a cross-section area of *A*, and a moment of inertia *I*. The structure is made of a material with a modulus of elasticity *E*. It is rigidly clamped on the left end and supported by two springs at a distance *L* from the left end and at the very right end, with the respective stiffness of k_1 and k_2. A load with a mass of M_1 is assumed to be applied vertically at a distance *L* from the right end. There is a *very small* gap ε between the loading position and a point contact that is fixed on the ground.



You are asked to analyze the structure using finite-element formulation.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down the element stiffness matrix and assembly stiffness matrix.
5. Determine the vertical deflection of the structure at the loading position, assuming that the structure does not touch the point contact. Show all steps to find results. No need to calculate the final values.
6. Now assume that with a larger load with a different mass M_2 attached in a second design and that the structure just touches the point contact. Determine the deflections at the two positions where the springs are attached. Show all steps to find results. No need to calculate the final values.

Element A - Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where *E*, *A*, and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; *l* and *m* are direction cosines.

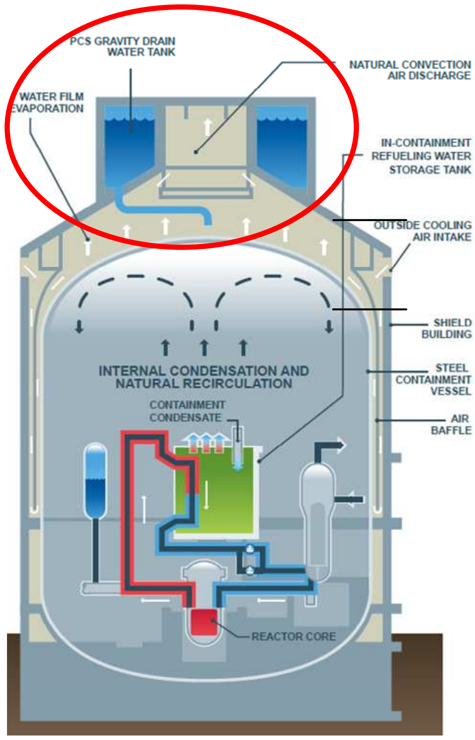
Element B - Stiffness Matrix

$$[K]=\frac{2EI}{h^3}\begin{bmatrix} 6 & -3h & -6 & -3h \\ -3h & 2h^2 & 3h & h^2 \\ -6 & 3h & 6 & 3h \\ -3h & h^2 & 3h & 2h^2 \end{bmatrix}$$

where *E*, *I*, and *h* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

3) Numerical Methods

A key characteristic of the new AP1000 nuclear reactor to be built in Augusta, Georgia, is that it will be cooled using a Passive Containment Cooling System (circled at the top in the figure on the right). This is in essence a large pool of water on top of the reactor housing that can be drained purely through gravity onto the steel containment vessel.



While designing this cooling system, you are asked to determine how long it will take for the tank to drain completely.

The flow rate of water through the pipe is characterized by the following equation:

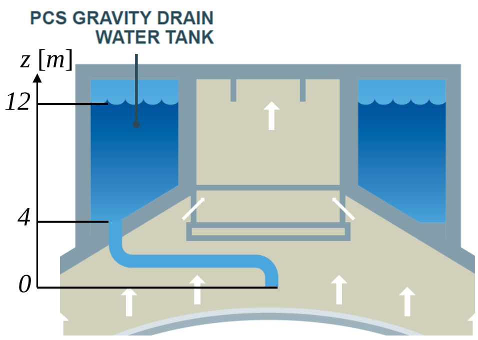
$$Q = A_{pipe}\sqrt{\frac{2zg}{1+fl}}$$

Assume that the volume of water in the tank is:

$$V = (z-4)A_{tank}$$

with an initial water height of $z = 12m$. The water volume decreases at the rate of Q, which is a function of the water height, z.

Variable	Value	Meaning
Q	see equation	Flow rate in $[m^3/s]$
V	see equation	Volume in tank in $[m^3]$
A_{tank}	370	Surface area of tank $[m^2]$
A_{pipe}	$\pi D^2/4$	Cross sectional area of pipe $[m^2]$
D	0.15	Diameter of pipe $[m]$
l	15	Length of the pipe $[m]$
f	0.0057	Friction factor of pipe [1]
g	9.81	Gravitational acceleration $[m/s^2]$
z	funct. of time	Water height $[m]$ (see figure right)



Questions: (relative grade allocations: a=60%, b=20%, c=20%)

a) Use RK4, to determine (approximately) how long it takes for the tank to empty.

$x_{n+1} = x_n + (h/6)(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4})$ with

$k_{n1} = f(t_n, x_n)$,

$k_{n2} = f(t_n + (h/2),\ x_n + (h/2)k_{n1})$,

$k_{n3} = f(t_n + (h/2),\ x_n + (h/2)k_{n2})$,

$k_{n4} = f(t_n + h,\ x_n + hk_{n3})$

b) Assume that the computational effort is characterized by the number of times you evaluate the derivative: $f(t,x)$. By how much (approximately) could you improve the accuracy of your answer if you were to double the effort (i.e., double the number of derivative evaluations)? Explain.

c) When solving this ODE problem using a state-of-the-art tool such as Matlab, one can typically choose from among multiple solution algorithms — not just RK4. Why? Why not just use one algorithm — the best algorithm — all the time? Explain.