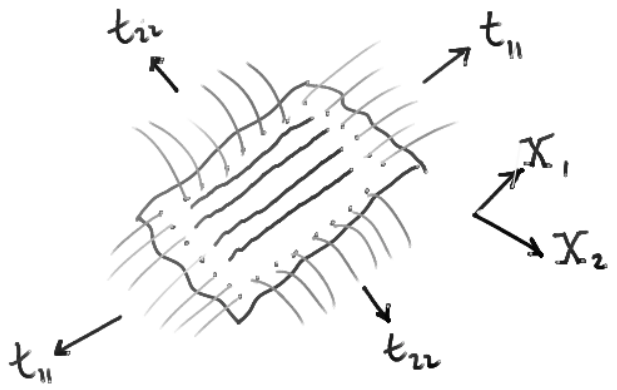


Consider biaxial stretching of a thin incompressible sheet of engineered myocardium, which to a first approximation is modeled as a transversely isotropic hyperelastic material with a preferred orientation due to fibrils running along the X_1-axis of the experimental setup. The strain-energy density function and first Piola-Kirchoff stress for the tissue is assumed to be, respectively:

$$W = C_1(I_1 - 3) + C_4 I_4^2$$
$$\mathbf{t} = \frac{\partial W}{\partial \mathbf{E}} \cdot \mathbf{F}^T - p\mathbf{F}^{-1}$$

where

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$
$$I_4 = \frac{1}{2}(\lambda_f^2 - 1)$$

are the three strain invariants, in terms of the relevant stretch ratios, λ_f is the fiber stretch, *p* is a Lagrange multiplier needed to enforce incompressibility, **E** is the Green strain tensor, and **F** is the deformation gradient tensor.

a) Show that the normal components of **t** can be written as follows:

$$\mathbf{t}_I = \frac{\partial W}{\partial \lambda_I} - p\lambda_I^{-1}$$

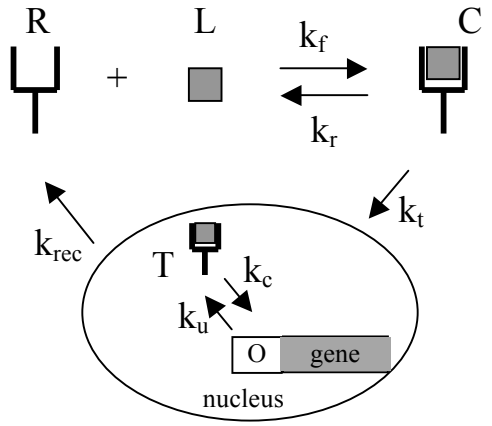
b) Given that $\lambda_2 = 1$ is held constant, derive an equation for the Cauchy stress components σ_1, σ_2, in terms of C_1, C_4, λ_f.

c) Again assuming $\lambda_2 = 1$, what are the behaviors of σ_1, σ_2 as λ_f grows larger?

Question 2

Consider the following model for steriod hormone signaling.

- Hormone L reversibly binds to its monovalent intracellular receptor to form the complex C.
- Signaling complex C translocates to the nucleus and acts as a transcription factor T. Translocation of the C to the nucleus is a first order reaction with k_t rate.
- T acts as a transcriptional activator binding to the operon O with coupling and uncoupling rates k_c and k_u, respectively. The complex of T and O is P and the rate of gene expression is proportional to the concentration of P with a rate constant k_s.
- T is recycled to the intracellular space following a first order reaction with k_{rec} rate.
- Assume no ligand depletion.



R = number of free receptors
L = free ligand concentration
C = number of complexes
T = number of hormone-receptor complexes in nucleus
O = number of unoccupied operons

R_T = total number of receptors/cell
O_0 = total number of operons

Derive the time-dependent system of governing equations for the rate of gene expression as a function of R_T, O_0, L and rate constants.

Question 3

A colleague in biology has generated a Protein X knockout mouse and would like take advantage of your biomechanics expertise to investigate whether Protein X plays a role in bone mechanics. To address this question, you decide to analyze the compressive mechanical properties of vertebral bodies in the knockout mice and wildtype control mice. The vertebral bodies consist of a thin cortical shell filled with trabecular bone. You find significant differences in the mechanical behavior of bones from Protein X knockout mice relative to wild type control mice. The force-displacement plots below are representative of these observed differences. Your colleague is intrigued and would like to know more about why the mechanical behavior of the knockout mouse bones is different.

A. For Zones I-IV of the Wildtype force-displacement curve, briefly describe what you think is happening physically to the specimen.

B. What parameters can you measure from the plots below? Describe the differences and similarities in mechanical behavior between the wildtype and knockout vertebrae.

C. Discuss all possible reasons for the observed differences in mechanical behavior.

D. What assumptions would you make about the material properties of the trabecular bone? Show mathematically the form of an appropriate constitutive model.