M.E. Ph.D. Qualifier Exam
Spring Quarter 1999

GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Quarter 1999

Bioengineering
EXAM AREA

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Assigned Number (DO NOT SIGN YOUR NAME)

- **Please sign your name on the back of this page—**

PLEASE ANSWER ALL THREE QUESTIONS. IF YOU ARE UNSURE OF A QUESTION, PLEASE STATE THE ASSUMPTIONS THAT YOU MADE IN ARRIVING AT YOUR ANSWER. PLEASE WRITE CLEARLY AND LEGIBLY.

1. Shown below is a uniformly thick planar sample of soft tissue loaded bi-axially as indicated by the arrows. The applied stress is uniform in x and in y.

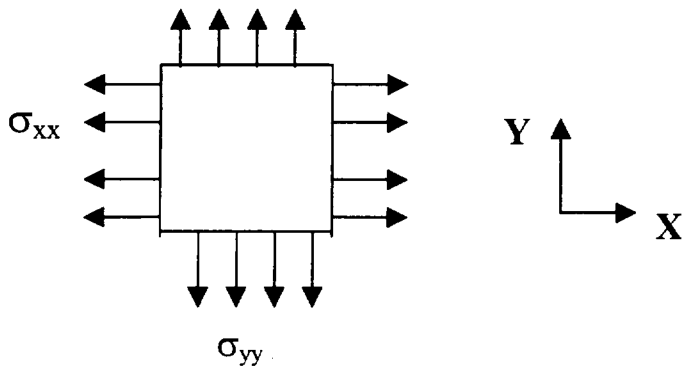
a) It is desired to measure the strain on the surface by tracking self-adhering particles. If the strain is uniform, what is the minimum number of particles that must be tracked to determine the strain? If the particles p_1, p_2..... are located at $(x_1,y_1)_0$.... and move to $(x_1,y_1)_1$.....as the tissue deforms, determine the extensional Green strain E_{xx}. Assume for the calculation that the minimum number of particles is present.

b) Sketch a plot of σ_{xx} vs E_{xx} for $\sigma_{yy} = 0$. In separate plot, show the effect of non-zero σ_{yy} on your plots of σ_{xx} vs E_{xx}. All stresses are tensile.

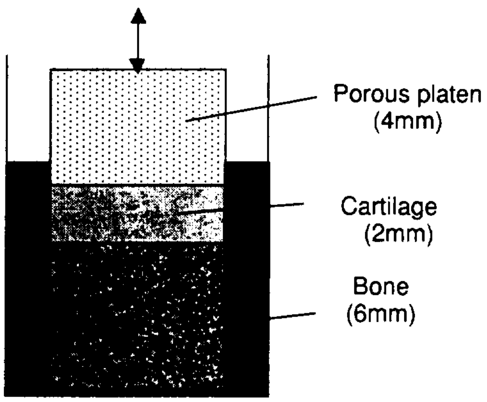
c) For an incompressible tissue, how does the thickness depend on the Green strain?

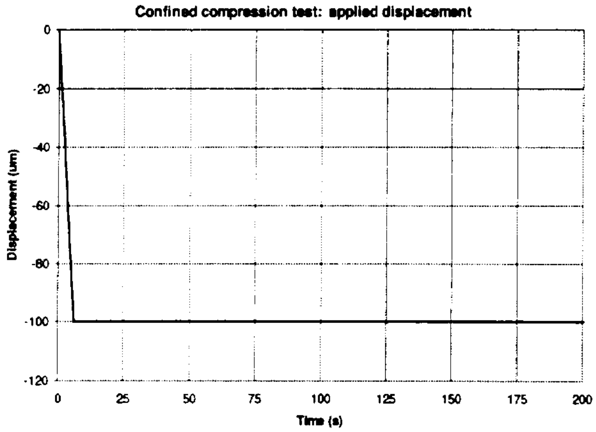
d) If the strain is non-uniform, discuss how would you measure it.

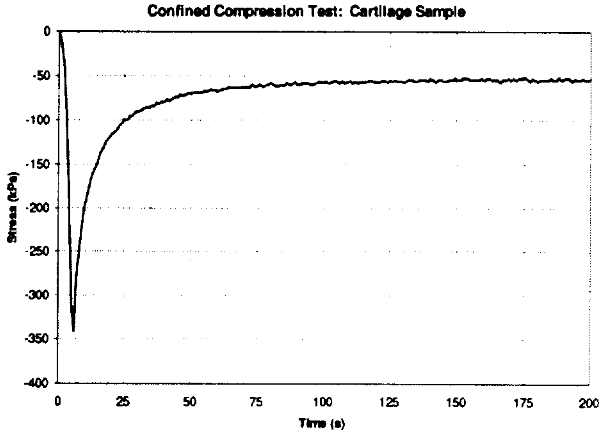
e) If the sample is isotropic, is there any shear strain? If the sample is orthotropic, is there any shear strain and if so, is the shear strain ever zero?



2. Consider the following "confined compression" test: An osteochondral core (6mm diameter) is removed from the patellofemoral surface of a young pig. The cartilage thickness is 2mm, and all but 6 mm of the subchondral bone is removed. The sample is placed in a stainless steel confining chamber and compressed under displacement control via a porous polyethylene platen (4 mm thick). The displacement is ramped to 100 μm compression in 6 seconds and held at that level. The stress calculated from the recorded axial load is shown below.







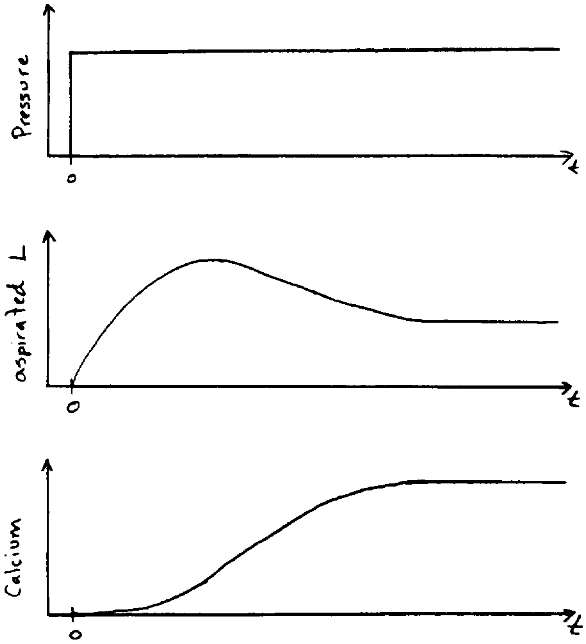
(a) Provide a physical interpretation for the observed behavior (both transient and equilibrium). How do these behaviors relate to the *in vivo* function of articular cartilage? What would change if we were testing a piece of osteoarthritic cartilage instead of healthy cartilage?

(b) Propose a simple mathematical model for this physical behavior. Discuss how to derive the relevant material parameters from these data and the physical significance of the parameters.

(c) Assume that the equilibrium behavior of the tissue can reasonably be modeled as linearly elastic. Calculate the equilibrium modulus, and determine an expression for this modulus in terms of any of the following commonly used material properties: Lamé constants, Young's modulus, Poisson's ratio.

(d) Suppose that we had carried out this experiment under *load* control instead of displacement control. (Assume that the ramp time and the stress as $t \rightarrow \infty$ are the same as in the original test). Sketch on the graphs above what you would expect the displacement and stress vs. time curves to look like. *Briefly* discuss why you expect this behavior.

3. A single fibroblast cell loaded with Fura-2 dye is subjected to a micropipet experiment. Fura-2 allows the intracellular calcium concentration to be measured in the cell in real time. During the micropipet experiment, the cell is subjected to a step change in pipet pressure. The following results are obtained from the experiment, where pressure is the pipet pressure, aspirated L is the length of the cell membrane aspirated into the pipet, and calcium is the average intracellular calcium concentration:



Part A) Wiesner et al. (PNAS, 1997) published a model of shear induced calcium influx in endothelial cells. These cells were exposed to fluid shear stress induced in one dimension in a flow chamber. Their model of the stretch-induced influx of calcium assumed that the stress has the following form: $\frac{\partial T_z}{\partial z} + (1-\varepsilon)\tau = 0$ and $T_x = 0$, where T_z was the tension in the direction of flow, τ was the fluid shear stress, ε was the load fraction borne by non-membrane structures, and T_x was the stress transverse to flow. Describe how the state of tension in the cell surface during the micropipet experiment differs from that of the cell in the flow chamber. How might this be related to the influx of calcium measured above? Use the membrane model from Cellular Biomechanics, where $\Delta P = \frac{\mu}{R_p}\left[\left(\frac{2L}{R_p} - 1\right) + \ln\left(\frac{2L}{R_p}\right)\right]$, to represent the micropipet experiments.

Part B) The data shown above indicate that the cell initially behaves viscoelastically in response to the step change in pressure. Does the complete response resemble that of a viscoelastic material? Please hypothesize on what may be occurring in the cell during the course of the experiment. Make references to specific portions of the figures above to support your reasoning.