GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2003

Bioengineering
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- Answer **three** of the four questions
- Please sign your **name** on the back of this page —

Please **p r i n t** your name here.

The Exam Committee will get a copy of this exam and will not be notified whose paper it is until it is graded.

Woodruff School of Mechanical Engineering
Bioengineering Qualifying Exam
Spring 2003

1. The pulsatile flow in an artery is to be modeled as a one-dimensional flow with a friction factor.

    a. Write down the form of this question

    b. An important parameter governing flow in blood vessels is the Womersley parameter (some times called the frequency parameter) defined as

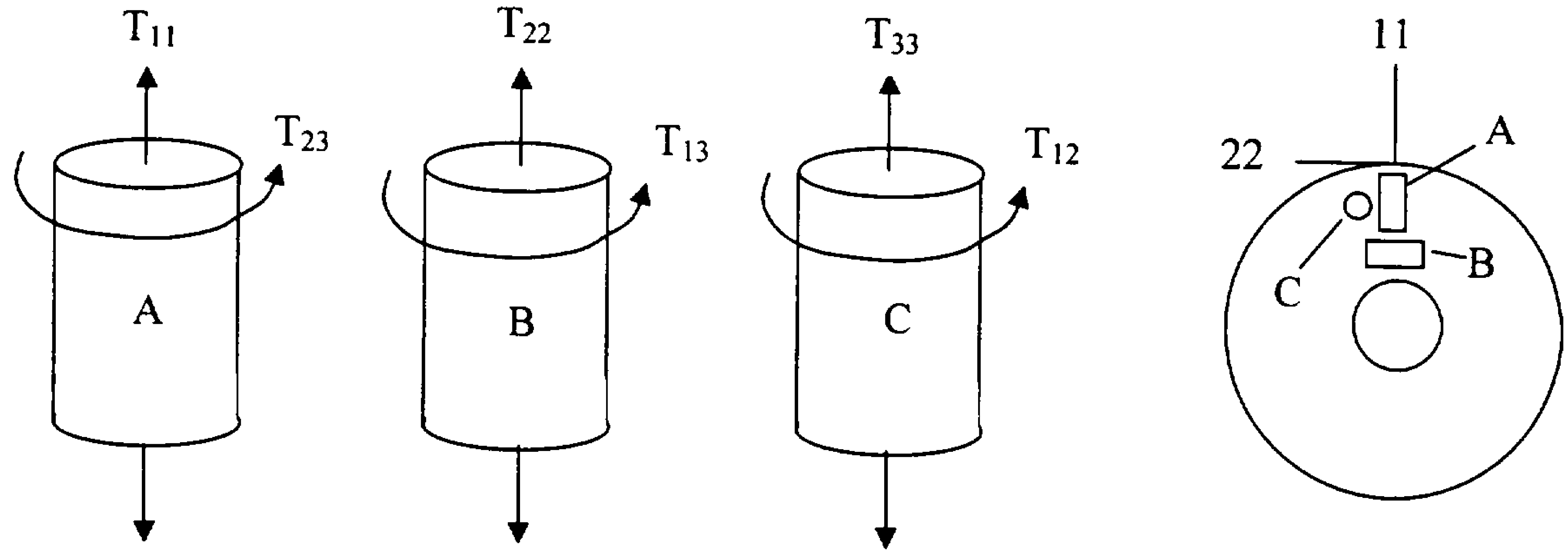
    $$\alpha = R\,(\omega/\upsilon)^{1/2}$$

    Where R is the vessel radius, ω is the frequency, and υ the kinematic viscosity.

    c. If $\alpha >> 1$, what is the limiting form of the equation in Part a?

    d. In the case of $\alpha >> 1$, sketch what the velocity profiles look like.

2. Three circular samples of cortical bone were machined from a region of a long bone assumed to be homogenous but not necessarily isotropic. The three samples were taken such that they aligned with respect to the radial (11), circumferential (22), and longitudinal (33) directions. Each sample was tested nondestructively in tension and torsion via the applied stresses T_{ij} shown below, resulting in the given strains E_{ij}.



Apply $\underline{T_{11} = 30\text{ MPa}}$	$\underline{T_{22} = 40\text{ MPa}}$	$\underline{T_{33} = 50\text{ MPa}}$
$E_{11} = 0.0026$	$E_{11} = -0.0020$	$E_{11} = -0.0013$
$E_{22} = -0.0015$	$E_{22} = 0.0035$	$E_{22} = -0.0013$
$E_{33} = -0.0008$	$E_{33} = -0.0011$	$E_{33} = 0.0029$
Apply $\underline{T_{23} = 40\text{ MPa}}$	$\underline{T_{13} = 30\text{ MPa}}$	$\underline{T_{12} = 40\text{ MPa}}$
$E_{23} = 0.0061$	$E_{13} = 0.0045$	$E_{12} = 0.0055$

A. Without knowing the values of the constants in the compliance matrix given below (i.e. based on the form of the matrix only), which of the following levels of material symmetry are <u>possible</u>? For each level of material symmetry, indicate the number of <u>independent</u> constants in K_{ij}.

<u>Circle all possible answers:</u> <u>Number of independent constants:</u>
Anisotropic
Orthotropic
Transversely isotropic
Isotropic

B. Fill in values for the elastic compliance matrix below based on the given cortical bone mechanical testing data from the three samples. What level of material symmetry most accurately describes the material behavior of the bone?

$$K_{ij} = \begin{matrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2G_{13} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2G_{12} \end{matrix}$$

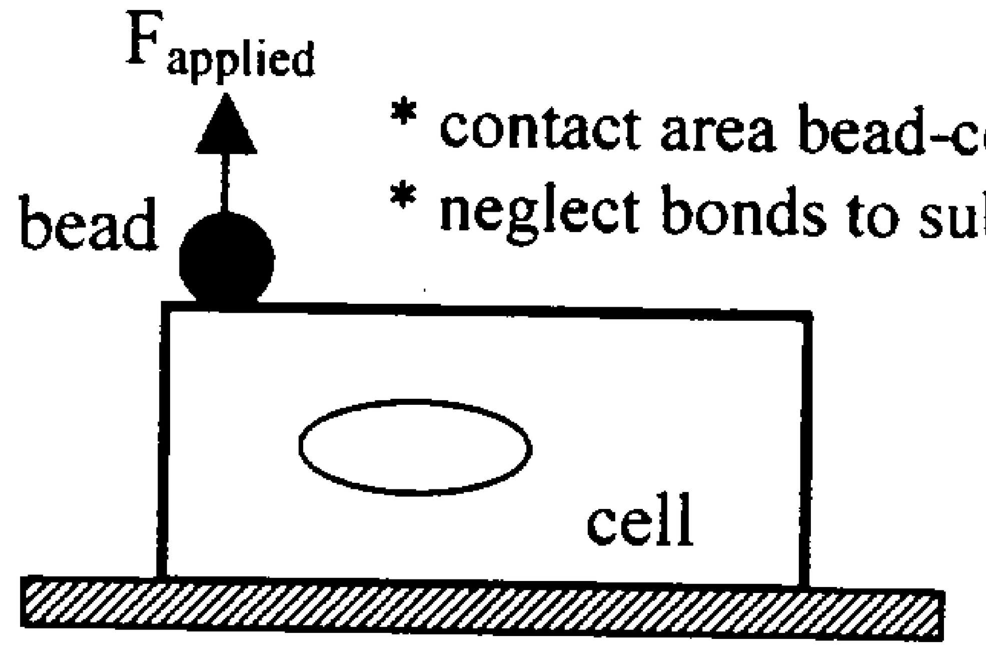
3. You are examining integrin-mediated adhesion by applying normal forces to fibronectin-coated beads using laser microtweezers (see Fig.). Recent studies indicate that the cytoskeletal protein vinculin interacts with bound integrins to strengthen adhesion. You hypothesize that fibronectin-integrin-vinculin complexes (complex 2) have higher rupture forces than fibronectin-integrin complexes (complex 1).

Based on the reaction diagram shown below for the fibronectin-integrin-vinculin interaction,

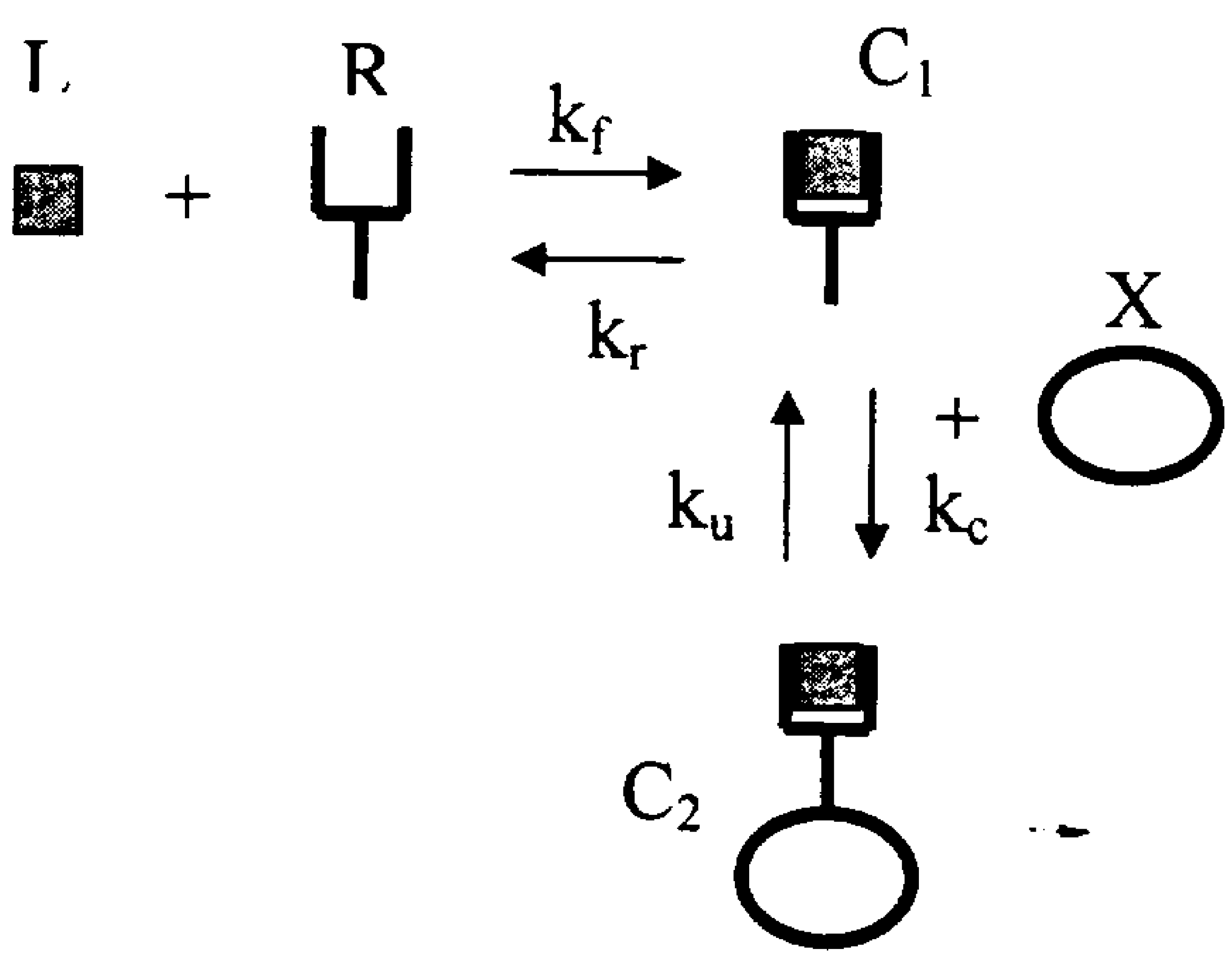
(i) Derive the time-dependent governing equations for C_1 and C_2 as a function of R_T, L_0, X_T, and rate constants. Assume no ligand depletion or integrin diffusion.

(ii) Assuming that adhesion strength is proportional to the number of complexes, provide estimates for the <u>steady state</u> values of $F_{applied}$ in terms of f_1. Assume $L_0 = K_D$ and $f_2 = 2f_1$.

- $k_c \gg k_u$ and $X_T \gg R_T$
- $k_c \gg k_u$ and $X_T \ll R_T$
- $k_c \ll k_u$ and $X_T \gg R_T$



* contact area bead-cell = 1% total cell area
* neglect bonds to substrate



R = number of <u>free</u> integrins/cell
C_1 = number of fibronectin-integrin complexes/cell
C_2 = number of fibronectin-integrin-vinculin complexes/cell
L = <u>free</u> fibronectin density
X = number of <u>free</u> vinculin/cell
R_T = total number of integrins/cell
X_T = total number of vinculin/cell
L_0 = total ligand concentration
forward bond rate = k_f
reverse bond rate = k_r
integrin-vinculin coupling rate = k_c
integrin-vinculin uncoupling rate = k_u
rupture force of single complex 1 = f_1
rupture force of single complex 2 = f_2