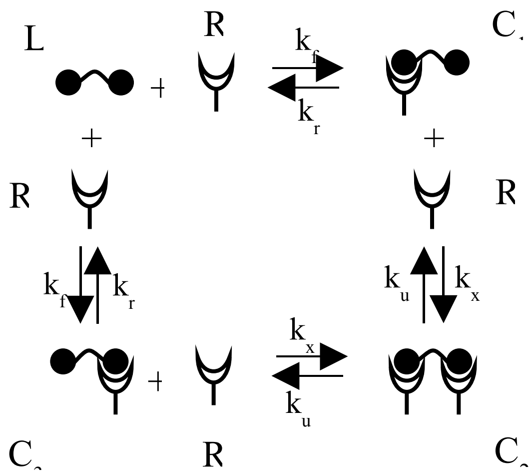
1. Consider this simple model for a bivalent ligand-monovalent receptor interaction.



R = number of free receptors
L = free ligand concentration
C_1 = number of receptor-left_ligand complexes
C_2 = number of cross-linked complexes
C_3 = number of receptor-right_ligand complexes

R_T = total number of receptors/cell

(a) Derive time-dependent governing equations for C_1, C_2, and C_3 as a function of R_T, L, and rate constants. Assume no ligand depletion.

(b) Rearrange solutions in (i) to obtain governing equations for singly occupied receptors ($C_s = C_1 + C_3$) and cross-linked receptors ($C_x = C_2$) as a function of R_T, L, C_s, C_x and rate constants.

2. (a)For the aortic valve of a human being fully open, describe the nature of the flow field both in words and using a sketch. Also, describe the process by which the aortic valve closes.

(b) Describe how you would analyze differences in aortic flow in a mouse as compared to that in a human.

3. The femur (thigh bone) consists of an outer shell of compact bone and an inner region of spongy bone. A man is standing on one foot and holding a heavy package. We can determine from experiment the following: the radius of the spongy region is r_s, the outer radius of the femur is r_c, the modulus of elasticity for the spongy bone is E_s, the modulus of elasticity for the compact bone is E_c, and the weight of the man and the package together is W.

(a) Find a relation for the stresses in the different regions of the femur in terms of r_s, r_c, E_s, E_c, and W. Is there a ratio of $E_s : E_c$ at which the contribution of the spongy bone toward the overall stress may be neglected?

(b) Assume next, that the man twists his body and applies a torque T on his femur. We can experimentally determine that the spongy bone has a shear modulus G_s and compact bone has a shear modulus G_c. What is the shear stress, $\sigma_{\theta z}$, in the spongy bone and the compact bone. Is there a ratio of $G_s : G_c$ at which the contribution of spongy bone toward the overall shear stress may be neglected?

(c) While the man is standing on one foot and twisting, assuming a state of plane stress at the outer surface of the bone (in the $\theta - z$ plane), what is the direction of the principal stresses? What the direction of maximum shear stress?

Axial Extension of a Rod

$$\sigma_{zz} = \frac{f}{A} \qquad u_z(z=b) - u_z(z=a) = \int_{z=a}^{z=b} \frac{f(z)}{A(z)E(z)}$$

Torsion of a rod or tube

$$\sigma_{z\theta} = \frac{Tr}{J} \qquad \Theta(z=b) - \Theta(z=a) = \int_{z=a}^{z=b} \frac{T(z)}{J(z)G(z)} dz$$

$$J = \frac{\pi c^4}{2} \text{ (for solid cylinder)} \qquad J = \frac{\pi (c^4 - d^4)}{2} \text{ (for hollow cylinder)}$$

Stress transformation equations for plane stress (in *x-y* plane)

$$\sigma_{x'x'} = \sigma_{xx} \cos^2\alpha + 2\sigma_{xy} \sin\alpha \cos\alpha + \sigma_{yy} \sin^2\alpha = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\alpha + \sigma_{xy} \sin 2\alpha$$

$$\sigma_{y'y'} = \sigma_{xx} \sin^2\alpha - 2\sigma_{xy} \sin\alpha \cos\alpha + \sigma_{yy} \cos^2\alpha = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{yy} - \sigma_{xx}}{2} \cos 2\alpha - \sigma_{xy} \sin 2\alpha$$

$$\sigma_{x'y'} = \sigma_{y'x'} = 2\left(\frac{\sigma_{yy} - \sigma_{xx}}{2}\right) \sin\alpha \cos\alpha + \sigma_{x'y'} \left(\cos^2\alpha - \sin^2\alpha\right) = \frac{\sigma_{yy} - \sigma_{xx}}{2} \sin 2\alpha + \sigma_{xy} \cos 2\alpha$$

$$\alpha_p = \frac{1}{2} \tan^{-1}\left[\frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}}\right] \qquad \alpha_s = \frac{1}{2} \tan^{-1}\left[\frac{\sigma_{yy} - \sigma_{xx}}{2\sigma_{xy}}\right]$$

Here σ_{ij} denotes the components of stress, $u_z(z)$ is the displacement at location *z* along the axis of the rod, *A*(*z*) is the cross sectional area of the rod at *z*, *E*(*z*) is the elastic modulus at *z*, *J*(*z*) is the second polar moment of area at *z*, $\Theta(z)$ is the angle of twist at *z*, *G*(*z*) is the shear modulus at *z*, *c* is the outer radius for the rod/tube, *d* is the inner radius of a tube, α denotes the angle between the *x* and x' coordinate axes, α_p is the angle of principal direction, and α_s is the angle of maximum shear stress.