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GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Fall Quarter 1995

Applied Math
EXAM AREA

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Assigned Number **(DO NOT SIGN YOUR NAME)**

-- Please sign your name on the back of this page --

PLEASE WORK ALL FIVE PROBLEMS

1a. Find the general solution of $y''+2y'+y = \exp(-3x)$.

b. Find the solution of

$$y = Ay, \quad y(0) = (3,0)^T \qquad A = \begin{bmatrix} 2 & -4 \\ 1 & -3 \end{bmatrix}$$

2. The central difference approximation for the second derivative of a function, f(x), is given by:

$$f''(x_i) \cong \frac{f(x_{i+1}) + f(x_{i-1}) - 2f(x_i)}{h^2})$$

Show that the error of this approximation is $O(h^2)$.

3. Consider two 3x3 matrices **W** and **D**. Assuming that **D** is symmetric and positive definite, and **W** is anti-symmetric. Please prove

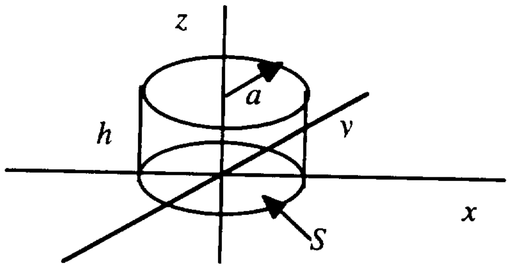
(i) $-\frac{1}{2}tr(\mathbf{W}\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1}) > 0$, where $tr(\mathbf{A})$ is the trace of **A**, e.g.,

$$tr\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = a_{11} + a_{22} + a_{33}$$

(ii) The three roots of $Det(\mathbf{W} + i\lambda\mathbf{D}) = 0$ are all real and they are given by

$$\lambda = 0 \text{ and } \lambda = \pm\left[-\frac{1}{2}tr(\mathbf{W}\mathbf{D}^{-1}\mathbf{W}\mathbf{D}^{-1})\right]^{1/2}.$$

4.



Consider the vector functions:

$$\vec{F}(\vec{r}) = 6z\hat{\mathbf{e}}_z$$
$$\vec{G}(\vec{r}) = 6z\hat{\mathbf{e}}_y$$

where $\hat{\mathbf{e}}_x, \hat{\mathbf{e}}_y, \hat{\mathbf{e}}_z$ are unit vectors in the x, y, and z diredctions respectively. Let S be a cylindrical surface of radius a and heigth h coaxial with the z axia with its base in the xy plane as shown in the figure.

a. Use the divergence theorem to evaluate $\oint\!\!\oint_S \vec{F}(\vec{r}) \cdot d\vec{A}$.

b. Verify the above result by evaluating $\oint\!\!\oint_S \vec{F}(\vec{r}) \cdot d\vec{A}$ directly. Show all work.

c. Use the divergence theorem to evaluate $\oint\!\!\oint_S \vec{G}(\vec{r}) \cdot d\vec{A}$.

d. Verify the above result by evaluating $\oint\!\!\oint_S \vec{G}(\vec{r}) \cdot d\vec{A}$ directly. Show all work.

e. Find the scalar function $\phi_F(\vec{r})$ such that $\vec{F}(\vec{r}) = \vec{\nabla}\phi_F$ or prove that such a function does not exist.

f. Find the scalar function $\phi_G(\vec{r})$ such that $\vec{G}(\vec{r}) = \vec{\nabla}\phi_G$ or prove that such a function does not exist

5. The solution to the equation $U_{tt} = c^2 U_{xx}$ with $U(0,t) = 0 = U(L,t)$, $U_t(x,0) = 0$, $U(x,0) = f(x)$ can be shown to be $U(x,t) = \Sigma b_n \cos\frac{n\pi ct}{L} \sin\frac{n\pi x}{L}$. Using the transformation $\xi = x + ct$, $\eta = x - ct$ show that above pde reduces to the form $U_{\xi\eta} = 0$.

Subsequently find the general form of its solution and show that it corresponds to the solution given above.