GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2001

Applied Math
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

Applied Math Qualifiers
Spring 2001

Answer 4 out of 5 questions

Question 1.

a). Write down the equation for a sphere centered at the origin of an orthogonal Cartesian coordinate systems (x_1, x_2, and x_3) with radius of R.

b) Find the normal vector of this sphere, express it in terms of x_1, x_2, and x_3.

c) Using the divergence theorem, evaluate the following integrals on the sphere:

$$\int_s x_1 x_2 ds \qquad \int_s x_2^2 ds$$

d) Based on the above, what can you say about the general integral on the sphere

$$\int_s x_i x_j ds \qquad\qquad i, j = 1,2,3$$

Question 2:

Let L be the linear space spanned by the three row vectors [1, 0, 1, 0], [1, 1, 3, 0] and [0, 2, 0, 1].

a. Determine the required value of x for the vector [2, -1, x, 2] to be an element of L.

b. Demonstrate that the three original vectors above do *not* constitute an *orthogonal* basis for L. Then, using the *Gram-Schmidt process*, construct such a basis.

Question 3:

Solve the following ordinary differential equation:

$$y'' - 4y' + 4y = x, \quad y(0) = 1$$

Here x is the independent variable and

$$y'' = \frac{d^2y}{dx^2} \text{ and } y' = \frac{dy}{dx}$$

Question 4:

Solve the boundary value problem $U_t = \kappa U_{xx}$ with $U(0,t) = 20$, $U(L,t) = 60$, $U(x,0) = 100$ using separation of variables. κ is a constant.

Question 5:

The data collected and given below is the deflection y of an elastic beam at various points x along the beam. For physical reasons it is known that the deflection is described by a 3[rd] order polynomial. The data is subject to some noise in measurement of the deflection y, but the position along the beam x is known without significant error. Establish the equations to find the point of maximum deflection by way of a Newton-Raphson approach to find the zero of a polynomial. Minimize the effect of the measurement noise in a least squares sense. The numerical answer is not requested. Your answer should define the equations that must be solved but not solve them.

x_i	0	1	2	3	4	5	6	7	8
y_i	0	0.40	0.80	0.94	1.00	0.93	0.70	0.40	0