GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2002

Applied Mathematics
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

- **Please sign your name on the back of this page—**

Ph.D. Qualifying Examination – Applied Mathematics – Spring, 2002

Work ALL problems

1.

Solve the following initial-boundary-value problem *by the method of separation of variables.*

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 \le x \le l, \ t > 0$$
$$u(0,t) = 0$$
$$\frac{\partial u(l,t)}{\partial x} = 0$$
$$u(x,0) = f(x)$$
$$\frac{\partial u(x,0)}{\partial t} = 0$$

where c is a constant and $f(x)$ is a given function. **Consider all possibilities for the sign of the separation constant.**

2.

Consider a spring-mass-damper system under no external force:

$$m\frac{d^2 y(t)}{dt^2} + c\frac{dy(t)}{dt} + ky(t) = 0,$$

This is a scalar ordinary differential equation in which t is the time (the independent variable), $y(t)$ is the displacement, $m > 0$ is the constant mass, $c > 0$ is the damping constant, and $k > 0$ is the spring constant. The initial conditions are

$$y(0) = k_1, \ y'(0) = k_2.$$

a) Obtain the displacement $y(t)$, $t \ge 0$. Depending on the relative magnitudes of m, c, and k, there are three possible cases that need to be addressed.

b) Based on the results of part a), discuss whether it is possible for all three cases to exhibit a solution $y(t)$ that oscillates (not necessarily with constant amplitude) about the point $y = 0$.

By this, we mean the following: assume that the initial displacement is strictly positive (that is, $k_1 > 0$); is it possible that the displacement $y(t)$ decreases to zero at a given time (that is, there is a $\bar{t} > 0$ such that $y(\bar{t}) = 0$), then becomes negative and reaches a minimum, and then increases (but needs not to be back to $y = 0$)?

3.

A second-order polynomial, $p(x)$, is to be determined that matches the following experimental data:

x_i	y_i
0	-1
1	0
2	2

In addition to the above data, assume it is known about the experiment that p must have a unit slope at 0: $p'(0) = 1$.

Find a second-order polynomial, $p(x)$, such that

- $p'(0) = 1$ *exactly,*
- $p(x)$ best matches the remaining conditions, $p(x_i) = y_i$ for $i = 0, 1, 2$, **in the sense of least squares.**

4.

a) Find the factors **L** and **U** corresponding to the **LU** decomposition of the matrix **A**:

$$\mathbf{A} = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

b) Find the inverse and the determinant of the matrix **B**:

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

c) Decide for or against the positive definiteness of the matrix **C**:

$$\mathbf{C} = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}.$$