Acoustics Ph.D. Qualifying Examination
Fall 2012
Closed book

Answer **all** of the following **three** questions.

1. A bar of 1 cm^2 cross-sectional area and 0.25 m length is free to move at x = 0 and is loaded with 0.15 kg at x = 0.25 m. The bar is made of steel (ρ = 7700 kg/m^3, Y = 19.5E10 Pa).
   a. Starting with the 1-D wave equation with displacement (ξ) as the dependent variable, calculate the fundamental frequency of longitudinal vibration of the mass-loaded bar. (*Hint:* tan(Z) = -0.78Z yields possible solutions of Z = 2.11, 4.97, 51.9, 67.6)
   b. Calculate the position at which the bar may be clamped to cause the least interference with its fundamental mode of vibration
   c. Calculate the ratio of the displacement amplitude of the free end to that of the mass-loaded end, when the bar is vibrating in its fundamental mode
   d. Calculate the frequency of the first overtone of this bar

2. The response of a generalized quadrupole receiver is given by

$$
\begin{aligned}
E(t) = {} & Q_{xx}\frac{\partial^2 p(\vec{R},t)}{\partial x^2} + Q_{xy}\frac{\partial^2 p(\vec{R},t)}{\partial x \partial y} + Q_{xz}\frac{\partial^2 p(\vec{R},t)}{\partial x \partial z} \\
& + Q_{yx}\frac{\partial^2 p(\vec{R},t)}{\partial y \partial x} + Q_{yy}\frac{\partial^2 p(\vec{R},t)}{\partial y^2} + Q_{yz}\frac{\partial^2 p(\vec{R},t)}{\partial x \partial z} \\
& + Q_{yx}\frac{\partial^2 p(\vec{R},t)}{\partial z \partial x} + Q_{yy}\frac{\partial^2 p(\vec{R},t)}{\partial z \partial x} + Q_{yz}\frac{\partial^2 p(\vec{R},t)}{\partial^2 z}
\end{aligned}
$$

For an incident plane wave, $p(\vec{R},t) = p_o f(\vec{n}\cdot\vec{R} - ct)$ where $\vec{n}$ is the direction of propagation and f is arbitrary:

a. Express the response of a lateral quadrupole (with $Q_{xy} = Q_o$ and all other Qs zero) in terms of the given variables and the spherical angles θ and Φ (θ is the angle with respect to the z axis and Φ is the azimuth angle.)

b. Express the response of a linear quadrupole (with $Q_{xx} = Q_o$ and all other Qs zero) in terms of the given variables and the spherical angles θ and Φ (θ is the angle with respect to the z axis and Φ is the azimuth angle.)

c. What combination of quadrupole moments would yield a response proportional to $\sin^2\theta$?

d. Show that a quadrupole with $Q_{xx} = Q_{yy} = Q_{zz} = Q_{0}$. is omnidirectional. How would such a receiver differ from a monopole?

3. A rigid plate with an orifice at its center is tightly fitted inside an air duct to attenuate sound. The walls of the duct are rigid and its diameter is 0.15 m. The air is at 20°C and 1 atm, and the acoustic excitation is axisymmetric (n = 0). Viscous damping effects are negligible. For axisymmetric excitation, the acoustic pressure inside the duct can be expressed as:

$$p(r,z,t) = \sum_{m=0}^{\infty} A_{0,m} J_0(k_r r) e^{j\left(\omega t - \sqrt{k^2 - k_r^2}\, z\right)}$$

The rigid wall boundary condition, $\frac{\partial p}{\partial r}|_{r=a} = 0$, results in $J_1(k_r a) = 0$ where $k_r = \alpha_{0,m}/a$ and $\alpha_{0,m}$ is the mth zero of J_1.

a. What is the upper limit on frequency to ensure that only plane waves propagate inside the duct? (*Hint:* The first three zeroes of J_1 are $\alpha_{0,0} = 0, \alpha_{0,1} = 3.832, \alpha_{0,2} = 7.016$.)
b. Assuming plane wave propagation, what are the acoustic boundary conditions that must be satisfied on each side of the orifice and what is the pressure transmission coefficient, **T**, for a continuous wave disturbance? (*Hint:* Treat the orifice as a short tube that is flanged at both ends.)
c. What diameter of the orifice is needed to provide a transmission loss (TL) of 20 dB at 1000 Hz?