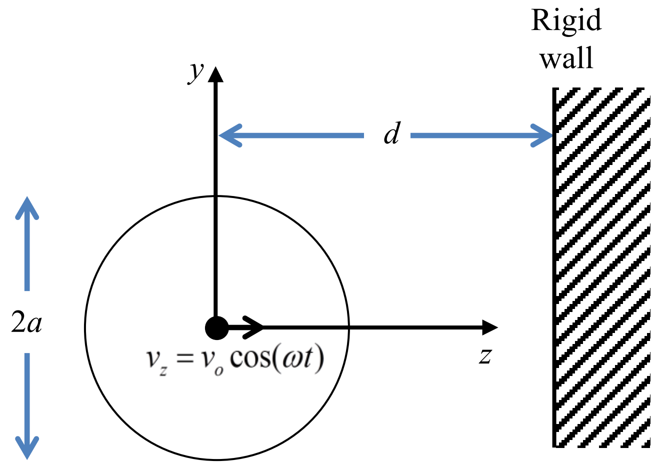
Acoustics Ph.D. Qualifying Examination
Spring 2012
Closed book

Answer all **three** questions.



1. An acoustically small, (ka <<1) rigid sphere of radius a is oscillating back-and-forth sinusoidally in the z direction with a velocity given by $v_z = v_o \cos(\omega t)$.

   a. Calculate the acoustic reaction force acting back on the sphere. Show that this force is mass-like and give an expression for the resulting radiation mass. **(5 points)**

   b. If the center of the sphere is at a distance, d, from a rigid immovable wall (kd << 1) with the direction of oscillation perpendicular to the surface, would you expect the radiation mass to be larger, smaller or the same as in the absence of the wall. Defend your answer**. (5 points)**

2. Consider the effect that gravity might have on ordinary acoustic propagation. Use the following three equations:
   - Linearized equation of state: $p = \rho_o c^2 s$,
   - Linearized continuity equation: $\partial s/\partial t + \vec{\nabla}\cdot\vec{u}' = 0$, and
   - Linearized Euler equation with gravity: $\partial\vec{u}'/\partial t + (1/\rho_o)\vec{\nabla}p = -gs\hat{z}$.

   Here, ρ_o, c and g (= the acceleration of gravity) are constants. (Note: "s", "p" and "u" are three variable representing respectively the local condensation, pressure and velocity of the fluid medium)

   a) (3 pts) Show that these equations can be reduced to: $\dfrac{1}{c^2}\dfrac{\partial^2 p}{\partial t^2} = \nabla^2 p + \dfrac{g}{c^2}\dfrac{\partial p}{\partial z}$

   b) (3pts) Write down the dispersion relationship. Hint: assume an oblique plane wave $\underline{p} = A\exp\{j(\omega t - \vec{k}\cdot\vec{r})\}$ with $\vec{k} = (k_x, k_y, k_z)$ and $\vec{r} = (x, y, z)$.

   c) (2 pts) Based on these results with $\hat{z}$ pointing vertically upward, should gravity affect acoustic waves that travel horizontally?

   d) (2 pts) For a 1000 Hz wave that travels upward in room temperature air, assume that $k_z \approx \omega/c$ and estimate the value of the ratio $\left(\dfrac{k_z g}{c^2}\right)\Big/\left(\dfrac{\omega^2}{c^2}\right)$. Comment on the significance of gravity for airborne acoustic waves at this frequency.

3. **(10 points)** Reflection from a normally reacting planar boundary:

    a. Beginning with the basic relationship: $z_n = r_n + ix_n$; derive the following expression for the sound power reflection coefficient ($\propto_r$) for a plane acoustic wave in air, impinging at oblique incidence on the surface of a normally reacting solid:

    $$\propto_r = \frac{(r_n cos\theta_i - \rho_1 c_1)^2 + x_n^2 cos^2\theta_i}{(r_n cos\theta_i + \rho_1 c_1)^2 + x_n^2 cos^2\theta_i}$$

    b. Now assume the solid is an acoustic tile panel with a normal specific acoustic impedance of $1000 - 1300i$ rayls. For a plane acoustic wave in air ($\rho_1 c_1 = 414$ rayls) impinging on the surface of the panel at 45 degrees, compute:

        i. the sound power reflection coefficient ($\propto_r$)

        ii. the sound power transmission coefficient ($\propto_t$)